

# FLIGHT FLUTTER TEST MODAL IDENTIFICATION BY USING NATURE INSPIRED ALGORITHMS: A CLASSICAL MATHEMATICAL MODELING

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**Abstract:** The aviation industry is coming under increasing pressure from governments, regulatory organizations and the general public to reduce emissions. To address this, the industry has come together and pledged to cut net carbon emissions to zero by 2050. To help meet this goal, EMBRAER is exploring a wide range of bold but viable aircraft designs in the Energia concepts, evaluating a range of sustainable concepts to carry up to 50 passengers, considering a number of energy sources, propulsion architectures and airframe layouts. In addition, the company is active member of FutPrint50 Project together with an international consortium of universities, SMEs and organizations to accelerate the technologies needed to deliver a hybrid-electric aircraft that could enter service in 2035-40. From aeroelastic standpoint, the Green Aviation will impose additional challenges in design, development and certification, mainly from lighter structures in unusual configurations, with multiple engines and more electrification. Consequently, it is important the evolution of aeroelastic industrial process in both prediction and testing. The present paper is focused on an innovative approach to Flight Flutter Testing modal identification by using Nature Inspired Algorithms to solve the nonlinear identification problem without using gradients. The performance of this innovative approach is evaluated based on theoretical benchmarks and real Flight Flutter Testing data. The results encourage the future use of more representative, adaptive or complex mathematical models during identification process.

## 1 INTRODUCTION

At the 77th IATA Annual General Meeting in Boston, USA, on 4 October 2021, a resolution was passed by IATA member airlines committing them to achieving net-zero carbon emissions from their operations by 2050. This pledge brings air transport in line with the objectives of the Paris Agreement to limit global warming to well below 2°C. To succeed, it will require the coordinated efforts of the entire industry (airlines, airports, air navigation service providers, manufacturers) and significant government support. It will require a combination of maximum elimination of emissions at the source, offsetting and carbon capture technologies [1].

Propulsion technology and structural evolutions are the key lever to reduce the emissions, representing 78% of IATA's strategy towards net zero CO<sub>2</sub> emissions. It include wider use of sustainable alternative fuels (SAF, electricity and hydrogen), unusual aircraft configurations,

ultra-efficient aerodynamic and more electrified design. However, regardless of the key lever or innovation used, the final aircraft should assure the equivalent level of safety of the actual aviation industry or higher.

To help meet this goal, EMBRAER is exploring a wide range of bold but viable aircraft designs in the Energia concepts [2], as shown in Figure 1. The Energia project is evaluating a range of sustainable concepts to carry up to 50 passengers, considering a number of energy sources, propulsion architectures and airframe layouts. In addition, the company is active member of FutPrint50 Project [3] together with an international consortium of universities, SMEs and organizations to accelerate the technologies needed to deliver a hybrid-electric aircraft that could enter service in 2035-40. All this to achieve the industry-wide goal of net carbon zero by 2050.

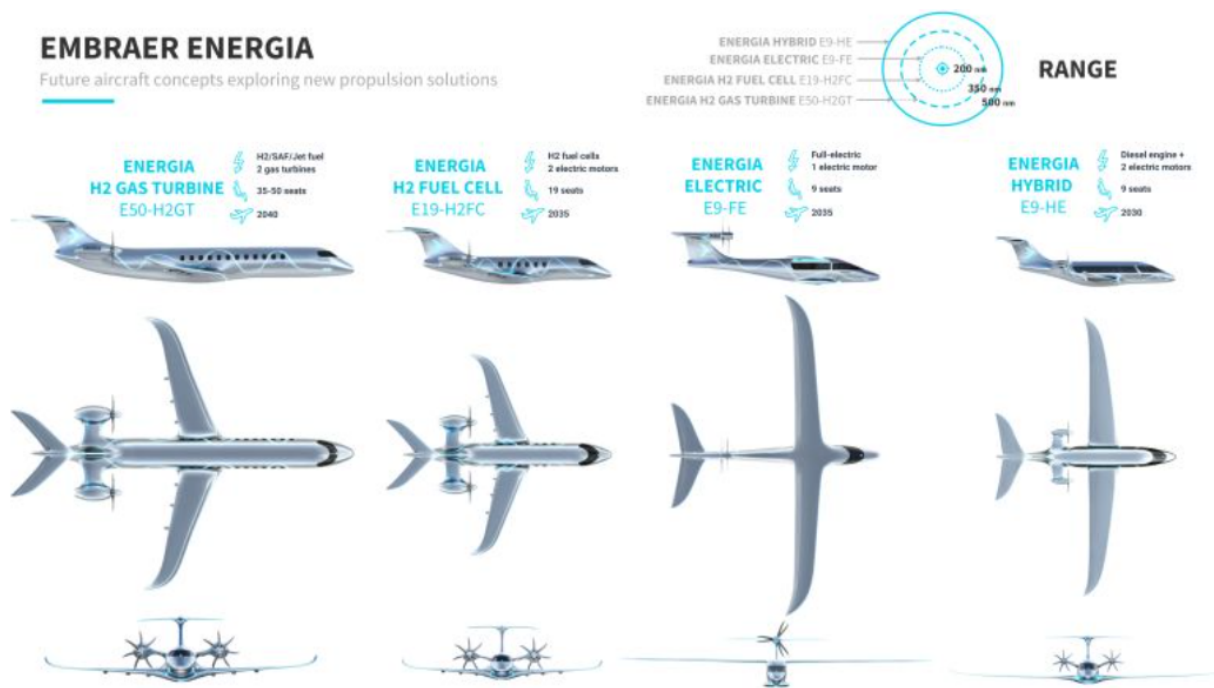


Figure 1: Embraer Energia Concepts.

From aeroelastic standpoint, the Green Aviation will potentially impose additional challenges in design, development and certification of these new aircrafts. Lighter structures in unusual configurations, with multiple engines and more electrification in terms of flight controls exemplify these potential future challenges.

Consequently, it is important the evolution of aeroelastic industrial process in both prediction and testing. The present paper is focused on an innovative approach to Flight Flutter Testing, specifically on data reduction process for modal identification. In addition, operational efficiency is a topic with paramount importance.

Flight Flutter Testing represents the biggest effort in experimental aeroelasticity and it is mandatory for the certification of a new aircraft. Modal parameters identification represents a critical task in demonstrating that the aircraft is free from aeroelastic instability within its flight domain.

There are different methodologies currently used for modal identification in both frequency and time domains, with positive and negative aspects in both cases. Frequency domain methods typically try to fit a specific mathematical model to the measured transfer function by adjusting

the model parameters, which requires the solution of a nonlinear problem.

Some linearization schemes were proposed in the literature to promote a first approximation to this nonlinear problem, such as multiplying the equation error by the denominator of the transfer function [4] or using the unknown denominator as a weighting function in an iterative scheme [5]. The linearization approach simplifies both mathematically and computationally the problem, but under the penalty of not minimizing the true error function and overemphasizing high frequency content, just to cite a few issues.

On the other hand, to classically solve the nonlinear minimization problem requires an initial estimate of the unknown transfer function parameters, as well as the first-order derivative information, which in turn may represent a hard or even unfeasible numerical task, specially when more complex mathematical model are adopted.

Conversely, Nature Inspired Algorithms (NIAs) do not require the derivative information, paving the way for the use of more complex or adaptive mathematical models in frequency domain, supplemented with more robustness to local minima. These algorithms are smart in the path to the optimum solution, with good numerical properties in general.

The present paper applies dissimilar NIAs - including Cuckoo Search, Differential Evolution, Whale, Grey Wolf, and Particle Swarm - to solve the nonlinear problem based on classical mathematical model in order to identify the modal parameters. The main goal is to verify the feasibility of the NIAs to solve the nonlinear problem.

The outline of the paper is as follows. Initially, the mathematical foundation of the identification problem is established. Next the Nature Inspired Algorithms are introduced and some of them selected to be applied to identification problem. Then the performance of the selected NIAs are evaluated based on theoretical benchmarks and real Flight Flutter Testing data.

## **2 MATHEMATICAL MODELING TO FLIGHT FLUTTER TESTING**

One of the important tasks in Flight Flutter Testing and subsequent data analysis is to reach an adequate aircraft response level for all vibration modes of interest. This can be satisfied by using appropriate onboard excitation devices with sufficient control on both magnitude and frequency bandwidth. Also, a robust flight test measurement array is fundamental for the acquisition of artificial excitation and subsequent vibratory response.

Industrial test experience indicates that data obtained generally contains considerable response level due to atmospheric operation, mainly promoted by turbulence and shock effects. Transient components are also present in the response due to aircraft's attitude changes, gust and shock waves, specially during dives and operation at high Mach number. Small response components are also promoted due to some systems operation, such as propulsion and hydraulics. The source of all those excitations are considered operational and they cannot be normally measured.

Figure 2A exemplifies a scenario where the vibratory response is generated fundamentally from the artificial excitation only, while Figure 2B presents a response due to both unmeasured excitation sources and measured artificial excitation. These different scenarios were observed during the same Flight Flutter campaign, but at different moments.

Beyond the excitation aspects, another relevant point to be observed in mathematical modeling are the assumptions about the characteristics of the aeroelastic system itself, specifically about linearity, temporal variance and initial conditions.

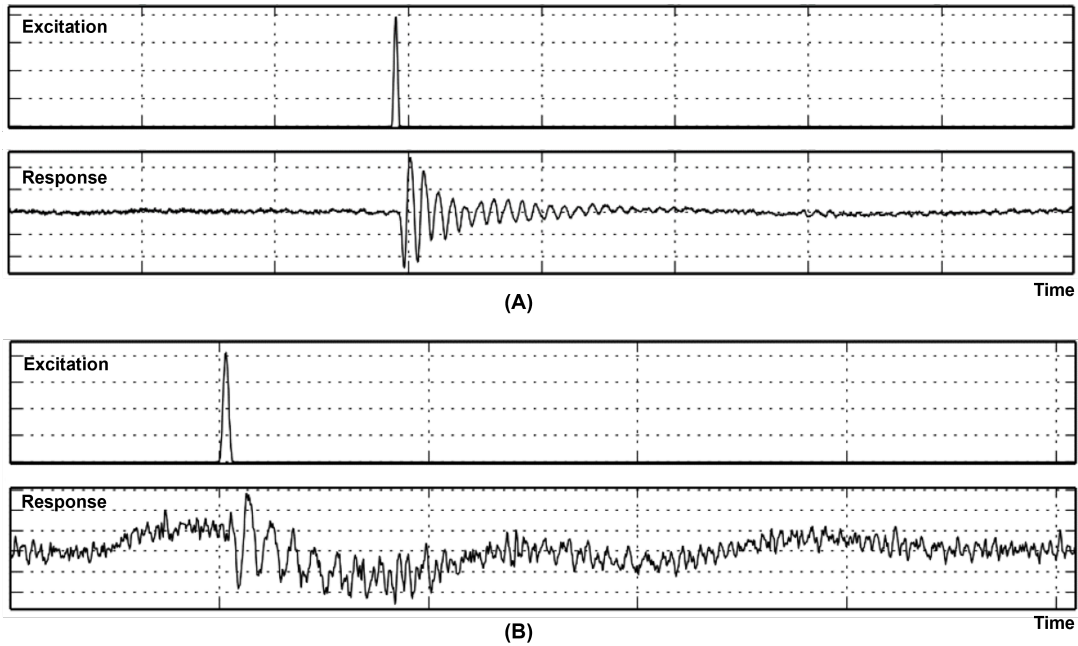


Figure 2: Different raw response scenarios observed during the same real Flight Flutter Test campaign: (A) Vibratory response is generated fundamentally from the artificial excitation and (B) Vibratory response generated by additional unmeasured excitation sources, in addition to the artificial excitation.

Linearity means that the relationship between the excitation  $x(t)$  and the response  $y(t)$ , both being regarded as functions, is a linear mapping: If  $a$  is a constant then the system response to  $ax(t)$  is  $ay(t)$ ; if  $\Delta x(t)$  is a different excitation with system response  $\Delta y(t)$  then the response of the system to  $x(t) + \Delta x(t)$  is  $y(t) + \Delta y(t)$ . This applies for all choices of  $a$ ,  $x(t)$ ,  $\Delta x(t)$ . In summary, the natural dynamic properties are unique and they do not change in function of the response state.

Time invariance means that whether we apply an excitation to the system now or  $T$  seconds from now, the response will be identical except for a time delay of  $T$  seconds. That is, if the response due to excitation  $x(t)$  is  $y(t)$ , then the response due to excitation  $x(t - T)$  is  $y(t - T)$ . Hence, the system is time invariant because the response does not depend on the particular time the excitation is applied.

The initial conditions define the response state at starting time. Therefore, null initial condition means that the system at starting time is at rest, with all response states equal to zero.

Considering both the excitation aspects and the characteristics of the aeroelastic system, the mathematical modeling may be wondering in four different complexity levels:

- **Low Complexity:** it assumes strong hypothesis about both the aeroelastic system and excitation. The system is assumed as linear, time-invariant with null initial conditions and all response is generated by measured excitations.
- **Medium Complexity:** it assumes strong hypothesis only on the aeroelastic system. The system is assumed as linear, time-invariant with null initial conditions and the response is generated by measured and unmeasured excitations.
- **High Complexity:** it does not assume hypothesis about the aeroelastic system, only on the excitation. It assumed that the response is generated by measured excitations only.
- **Extreme Complexity:** it does not assume any hypothesis about the aeroelastic system

neither the excitation.

Beyond excitation aspects and the characteristics of the aeroelastic system, three topics are fundamental in terms of mathematical modeling: (i) the model domain and; (ii) type of input and; (iii) electronics noise.

- **Model Domain:** It establishes the domain in which the mathematical model is defined, typically time-domain, frequency domain or time-frequency domain.
- **Type of Input:** It establishes the type of input data used, typically time-histories, correlations, Power Spectral Densities (PSDs), Positive-Spectrum or Frequency Response Functions (FRFs).
- **Electronics Noise:** It establishes the presence of noise from the electronics of the measurement array in both excitation and response data.

## 2.1 Classical Mathematical Modeling

The present paper will adopt the low complexity level, mainly because the focus is on innovative approach to solve the associated identification problem by means of Nature Inspired Algorithms.

In addition, the frequency domain was selected to allow focus solely on the frequency band of interest. Moreover, the amount of data to be processed is also greatly reduced, resulting in improved computation times. FRF was selected as input data.

The combination of low complexity level in frequency domain using FRF was denominated herein as **Classical Mathematical Model**.

Putting focus on the input data, it is interesting to observe that FRFs can be matematically described as a rational function and expressed in different forms, including:

- Polynomial form, see Eq.1
- Pole-zero form, see Eq.2
- Pole-residue form, see Eq.3
- State-space form, see Eq.4

$$H(s, P) = \frac{\alpha_0 + \alpha_1 s + \cdots + \alpha_n s^n}{\beta_0 + \beta_1 s + \cdots + \beta_d s^d} \quad (1)$$

$$H(s, P) = k \frac{\prod_{m=1}^N (s - z_m)}{\prod_{m=1}^N (s - a_m)} \quad (2)$$

$$H(s, P) = \sum_{m=1}^N \frac{r_m}{s - a_m} + r_0 \quad (3)$$

$$H(s, P) = \mathbf{c}^T (s\mathbf{I} - \mathbf{A})^{-1} \mathbf{b} + r_0 \quad (4)$$

The polynomial form is the most classical and common form employed in the identification of modal parameters and this form was used in several traditional approaches such as in Levy [4], Sanathanan and Koemer [5], Richardson [6] among others.

Differently, in the present paper was considered advantageous the FRF description in terms of pole-residue because the poles appears explicitly on the denominator, making it easier for NIAs to operate.

Therefore, the mathematical problem to be minimized is described by Equation 5, which represent the absolute error between the experimental FRF and FRF model in terms of pole-residue,

$$\ell = \sum_{m=1}^{N_o N_i} \sum_{k=1}^{N_f} \left| H_m(\omega_k) - \left\{ \sum_{j=1}^N \left( \frac{A_{jm}}{j\omega_k - \lambda_j} + \frac{A_{jm}^*}{j\omega_k - \lambda_j^*} \right) + LR + (j\omega_k)^2 UR \right\} \right| \quad (5)$$

with  $N_f$  the number of the frequency lines,  $N_o$  the number of the measured outputs and  $N_i$  the number of the measured inputs. The parameter  $A$  represents the residuals,  $N$  the order,  $\lambda$  the poles and  $H_m(\omega_k)$  the measured FRF in terms of acceleration. In addition, UR and LR are the upper and lower residual terms, important to reduce the effects of out-of-band dynamics.

The norm-1 was selected for the error computation in Equation 5 because it is less sensitive to outliers (noise in FRFs) than norm-2. The possibility of using different non-linear objective functions may be beneficial for optimization process and it can be explored by using NIAs.

In order to reduce the computational time taken by the NIA, the minimization task consists in three basic steps : (i) first the poles are guessed by the NIA algorithm, (ii) the residuals  $A$  and the UR and LR terms are obtained by a linear least-squares procedure and (iii) the objective function described by 5 is evaluated. This process is repeated recursively until convergence criteria is met.

The practical drawback of adopting the Classical Mathematical Model is the need to reduce the effect of operational conditions and associated unmeasured excitation sources on the vibratory response of the aircraft by means of test execution restrictions and/or data pre-processing.

In a real test campaign, the inclusion of test restrictions - such as no test execution when turbulence level is significantly - may reduce the efficiency of the campaign impacting negatively its costs. These impacts encourage the future use of more representative, adaptive or complex mathematical models for the identification process.

### 3 NATURE INSPIRED ALGORITHMS

Nature inspired algorithms are gradient-free optimization solvers which are based on notorious biological or physical phenomena found in nature.

These algorithms do not require a complete description of the optimization problem. The only demand is the objective function to be minimized. As such, they are useful tools for solving non-linear and complex optimization problems with a plethora of local minima.

According to Mirjalili and Lewis [7], they can be categorized into three distinct groups: evolution-based, physics-based, and swarm-based methods.

Evolution-based are inspired on evolution theory, by which the fittest individuals of a given population are more likely to pass on their genes to the next generations. In the mathematical optimization context, the fittest individuals are those which present the lowest values of the objective function. The initial population is randomly generated and it evolves according to mating, reproduction, crossover and mutation mechanisms to diversify the population along iterations, with the fittest individuals being more likely to generate offspring and to persist through iterations. Genetic Algorithms (GA) [8], which are based on Darwinian evolution, are arguably the most popular evolution-inspired optimization algorithms. Other examples are Differential Evolution (DE) [9], Evolution Strategy (ES) [10] and Biogeography-Based Optimizer (BBO) [11].

Physics-based optimization algorithms, on the other hand, are inspired by physical laws or space theories. The most well-known algorithm of this class is perhaps the Simulated Annealing (SA) [12], which is based on thermodynamic laws that drive the annealing process in metallurgy. There are also several algorithms based on space theories, such as the Gravitational Local Search (GLSA) [13] and the Big-Bang Big-Crunch (BBBC) [14], to cite a few.

The swarm-based are typically inspired by the social behavior of animals or insects. Two of the most prominent algorithms in this class are the Particle Swarm Optimization (PSO) [15] and the Ant Colony Optimization (ACO) [16]. The PSO takes inspiration in the flocking behavior of birds: this gathering of individuals travel, forage or migrate in a collective fashion, constantly reacting to the environment. In the mathematical optimization context, this flock of birds is driven by the lowest objective function value in the search domain. Meanwhile, the ACO is based on the foraging behavior of ants: each individual of the colony searches for a food source (i.e., best solution) while leaving pheromones in its path, to communicate others of its findings. It has been famously used to solve the NP-hard traveling salesman problem. Other popular algorithms include the Cuckoo Search Algorithm (CS) [17], the Whale Optimization Algorithm (WOA) [7] and the Grey Wolf Optimization Algorithm (GWO) [18].

Regardless of their inspiration, all population-based (evolutionary and swarm) search for the global minimum of the objective function by generating multiple solutions (individuals) and modifying them according to the population characteristics. In the first search phase, to avoid being stuck at a local minimum, these algorithms encourage a random behavior from individuals, in what is called the exploration phase. Once the search domain has been surveyed, the individuals focus on the most promising areas in search of the global minimum, in what is known as the exploitation phase. An adequate balance between exploration and exploitation phases is of paramount importance for the correct convergence of population-based optimization algorithms.

One can also find in the literature metaheuristics based on human behavior. Examples are Tabu Search (TS) [19], Teaching Learning Based Optimization (TLBO) [20], Harmony Search (HS) [21], Imperialist Competitive Algorithm (ICA) [22], among others.

Table 1 collects some Nature-Inspired Optimization Algorithms for reference<sup>1</sup>. They are categorized according to the respective inspiration, class and year of proposal.

In order to give a deeper inside about the Nature Inspired Algorithms, one of the most popular will be presented in more details in the next section.

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<sup>1</sup>It represents a small sample of algorithms, for more comprehensive review see [23]

Table 1: Sample of nature-inspired algorithms developed over the years.

ALGORITHM	INSPIRATION	CLASS	YEAR OF PROPOSAL
SA [12]	Metallurgy Process	Physics	1983
GA [8]	Natural Selection	Evolution	1992
PSO [15]	Bird Flocking	Swarm	1995
DE [9]	Natural Selection	Evolution	1997
GLSA [13]	Gravitational Attraction	Physics	2003
ACO [16]	Ant Colony	Swarm	2006
MS [24]	Monkey	Swarm	2007
CS [17]	Cuckoo	Swarm	2009
EHO [25]	Elephant	Swarm	2015
WOA [7]	Whale	Swarm	2016
DSOA [26]	Dolphin	Swarm	2016
MSSA [27]	Salps Swarming	Swarm	2017
EPC [28]	Emperor Penguins Colony	Swarm	2019
SFO [29]	Sailfish	Swarm	2019
SSA [30]	Squirrel	Swarm	2019
GEO [31]	Golden Eagle	Swarm	2021

### 3.1 Cuckoo Search Algorithm

Cuckoo Search algorithm was proposed by Xin-She Yang and Suash Deb in 2009 [17]. It has been successfully applied to solve various optimization problems, such as speech reorganization, parameter tuning, and image processing but no direct application for parameter identification was found in the literature by the present authors. The following explanation is based on the work of Xin-She Yang [32].

It is based on the distinctive reproduction strategy of some species of this bird family. Instead of laying their eggs on a self-built nest, female cuckoos will do it on other species nests. The cuckoo hatchling will eventually be brought up and fed by birds of this other species, as if it was a chick of their own. Figure 3B illustrates this behavior.



Figure 3: The brood parasitism behavior of cuckoo birds: (A) Cuckoo lays its egg (the bigger one) in the nests of other bird species and (B) The host birds raise the cuckoo chick as their own.

In case the host bird spots the cuckoo egg, it decides between removing this alien egg from the nest or building a new one in another area. The identification task can be a challenging one, however: some cuckoo species are skilled at mimicking the appearance of the host birds eggs (see Figure 3A).



Another characteristic of this brood parasitism behavior is that the cuckoo egg usually hatches earlier than host ones: in such cases, the cuckoo chick will often toss the host eggs off the communal nest, therefore increasing its share of food and, thus, its own chance of survival.

These cuckoo traits form the basis of the optimization algorithm, which adopts the following assumptions [32]:

- Each cuckoo lays one egg at a time, and dump its egg in randomly chosen nest;
- The best nests with high quality of eggs will carry over to the next generations;
- The number of available host nests is fixed, and the egg laid by a cuckoo is discovered by the host bird with a probability  $p_a \in [0, 1]$ . In this case, the host bird can either throw the egg away or abandon the nest, and build a completely new nest. For simplicity, this last assumption can be approximated by the fraction  $p_a$  of the  $n$  nests are replaced by new nests (with new random solutions);
- Each egg in a nest represents a solution, and a cuckoo egg represent a new solution, the aim is to use the new and potentially better solutions (cuckoos) to replace a not-so-good solution in the nests.

The perturbation of the current solutions, as well as the location of the newly built nests are all driven by Lévy Flights. Named after the french mathematician Paul Lévy, the Lévy Flight is a random walk characterized by straight-line paths punctuated by sharp 90 degree turns. Some birds and insects exhibit this behaviour in their movement. By allowing both long and short movements, this process is adequate for both exploration and exploitation search phases. The greedy selection strategy ensures the best solutions are always kept from generation to generation.

This algorithm can be extended to the more complicated case where each nest has multiple eggs representing a set of solutions. For this present work, we will use the simplest approach where each nest has only a single egg.

The pseudo code of the Cuckoo Search algorithm for minimization problems is reproduced from [32] in Algorithm 1. Numerically functional codes can be found in the literature in different languages as Python and Matlab. In addition, several evolutions of this original algorithm can also be found.

### 3.2 NIAs Selection

Motivated by the No Free Lunch Theorem<sup>2</sup> [33], six NIAs were selected for further evaluation to minimize the nonlinear error function described by Equation 5.

The selection criteria was based on engineering judgment considering, among others, the simplicity of implementation, expected number of flops and similar applications found in the literature. For example, the PSO was included based on the similar work developed by Elkafafy et al. [34], meanwhile the DE was selected by its simplicity and low computational effort.

Table 2 presents the selected algorithms<sup>3</sup>, including some details about them.

<sup>2</sup>NFL theorem basically states that there is no single algorithm that can be most efficient to solve all types of problems.

<sup>3</sup>The algorithms GWO and WOA were implemented considering improved and modified versions of the originals.

**Algorithm 1** Pseudo code of the Cuckoo Search

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**Require:** Objective Function  $f(\mathbf{x})$ ,  $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$   
**Require:** Generate initial population of  $n$  host nests  $\mathbf{x}_i$   
**while**  $t < MaxGeneration$  or  $StopCriterion$  **do**  
    Get a cuckoo randomly to generate a solution by Lévy flights and evaluate its fitness  $F_i$   
    Choose a nest among  $n$  (say,  $j$ ) randomly and evaluate its fitness  $F_j$   
    **if**  $F_i < F_j$  **then**  
        Replace the solution in the nest  $j$  by the new solution  $i$   
    **end if**  
    A fraction ( $p_a$ ) of worse nests are abandoned and new ones are generated  
    Keep best solutions (or nests with quality solutions)  
    Rank the solutions and find the current best  
**end while**  
Postprocess results and visualization

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Table 2: Selected nature-inspired algorithms for Flight Flutter Testing application.

ALGORITHM	INSPIRATION	CLASS	YEAR OF PROPOSAL
PSO [15]	Bird Flock	Swarm	1995
DE [9]	Genetic	Evolution	1997
FFLY [32]	FireFly	Swarm	2007
CS [17]	Cuckoo	Swarm	2009
GWO [18]	Wolf	Swarm	2014
WOA [7]	Whale	Swarm	2016

The selected algorithms were in-house implemented using Python language and validated by using optimization benchmark functions, including Rosenbrock, Beale and Booth 2D functions. Once the correct implementation was verified, the algorithms were cleared to the next evaluation phase represented by real identification problem.

#### 4 PERFORMANCE EVALUATION FOR SELECTED NIAS

All the six algorithms presented in Table 2 were evaluated based on three different phases in evolutionary way. First and second phases are based on a theoretical benchmark, which represents the aeroelastic behavior of an aircraft and comply with all hypothesis of classical mathematical modeling. Phase three represents a real aircraft during its Flight Flutter Testing.

The purpose of the first phase is the evaluation of both computational performance and capability to find the minimum of objective function. In this phase, all six selected NIAs were used to solve the same problem, which is characterized by a single test condition. Only the two best NIAs will pass to the second phase.

In the second phase, the aeroelastic behavior is changing as a function of control parameter (as for instance, dynamic pressure) in order to simulate different scenarios, such as modal coupling and unbalanced damping. This represents a hypothetical aeroelastic modal evolution. The purpose of this phase is verify the robustness of two best NIAs in different and more difficult aeroelastic scenarios. Only one will pass to the last phase.

The purpose of third and last phase is final verification of the capability of the best NIA in a real Flight Flutter Testing case. In this phase not all hypothesis of classic mathematical modeling

are satisfied completely.

All algorithms were evaluated as similarly as possible, by using the same population size, maximum number of iterations, initialization type, among others.

#### 4.1 Theoretical Benchmark Evaluation

The theoretical benchmark represents the evolution of aeroelastic behavior described by dimensionless natural frequency and modal damping as function of a control parameter (for instance, dynamic pressure) for three vibration modes. Figure 6 shows the benchmark characteristics.

Spotchecks at ten different conditions were taken and used in the theoretical investigation. They were designed in order to represent typical aeroelastic behavior, including high modal coupling, unbalanced damping, hump mode and even close to flutter condition.

The spotcheck No.1 was considered for the first phase evaluations. It represents the simplest dynamic system, characterized by 3 spaced modes and balanced modal damping. Low level of electronic noise was included directly in real and imaginary parts of FRF in order to increase the representativeness of the problem. For simplicity, only one FRF was used.

Figure 4 presents the convergence behavior for all six selected NIAs. It can be immediately observed that four of them reach the minimum value of objective function (CS, DE, WOA and GWO) but two fail (PSO and FFLY). The non-convergence of PSO<sup>4</sup> and FFLY was considered sufficient for disqualification of both algorithms for the present application.

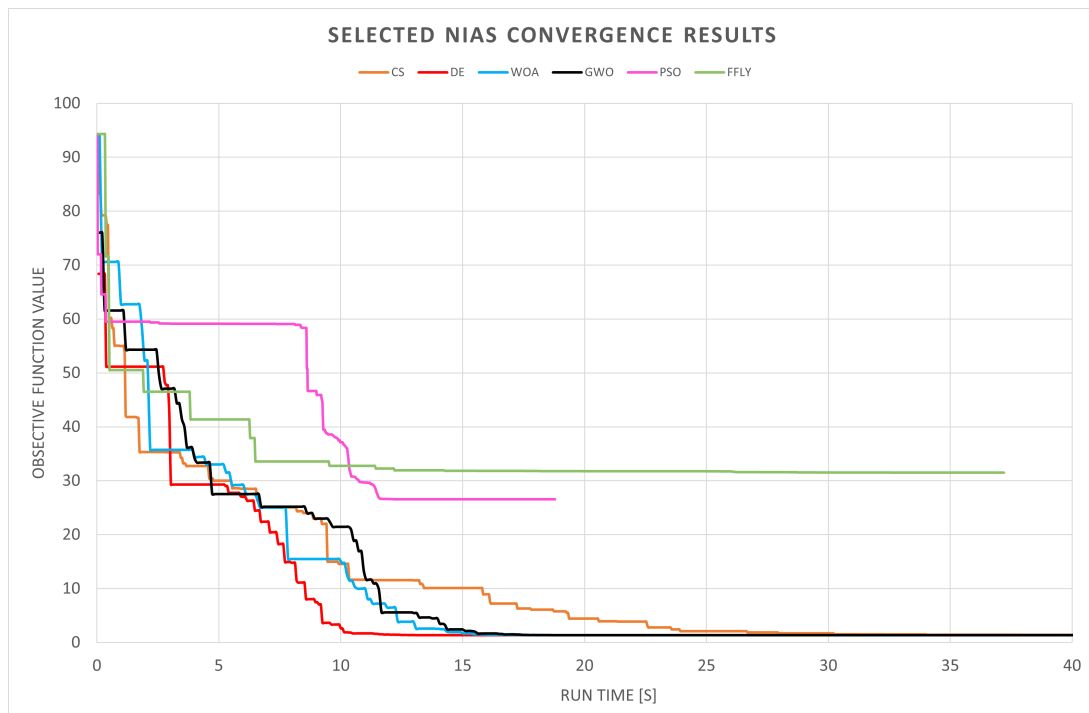


Figure 4: Convergence behavior for all six selected NIAs for snapshot No.1

The four convergent algorithms found the minimum of objective function with different run times. The fastest<sup>5</sup> one was DE with 11.7s to found the minimum, followed by WOA with

<sup>4</sup>Further investigation has shown that the convergence of PSO is assured by doubling the number of particle, but under the penalty of increase on the computational time.

<sup>5</sup>At this moment, the focus was on the convergency aspects not in the total computational time.

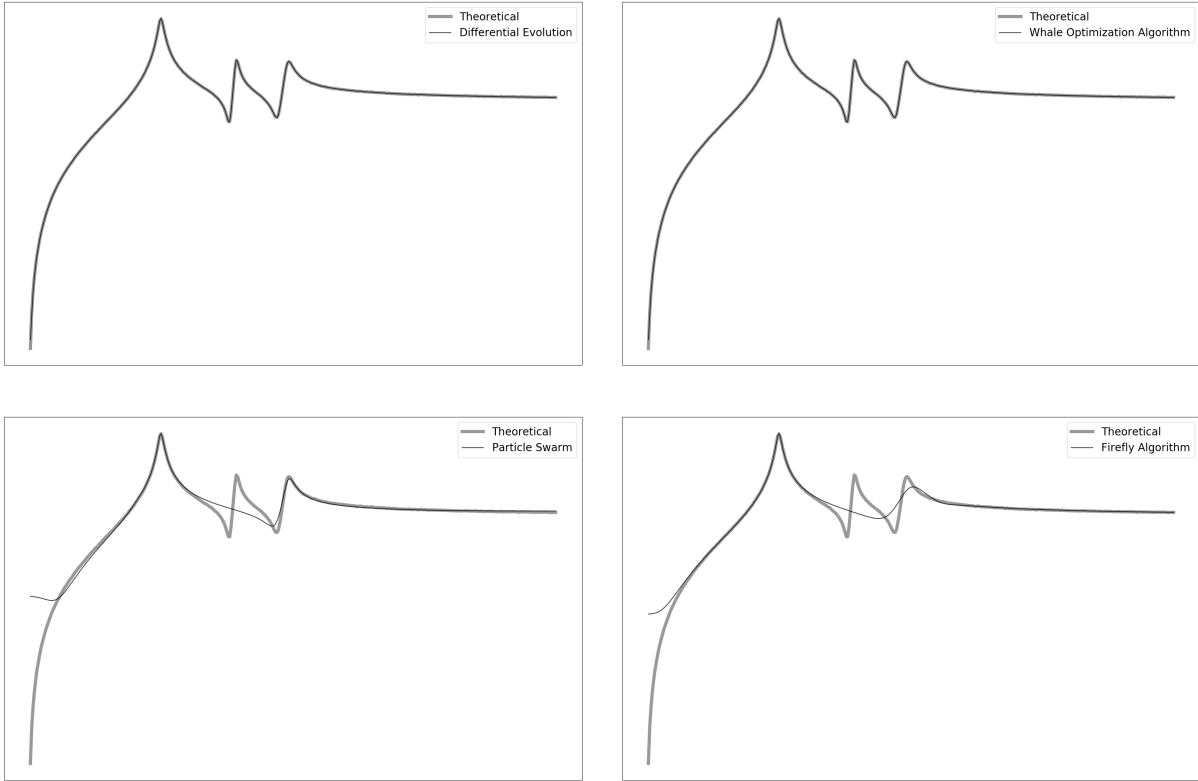


Figure 5: Final Fitting Results for the two best algorithms (DE and WOA) and two worst (PSO and FFLY). FRF represented in Bode diagram, magnitude only.

15.9s. The third one was GWO with 17.4s and the slowest the CS with 32.3s.

Table 3 presents<sup>6</sup> the identified modal parameters<sup>7</sup> for all three modes, including the exact values used in the simulation. Except for the disqualified ones (PSO and FFLY) all the other algorithms presented very similar results with negligible differences among them. In addition, they demonstrated good precision, with slightly error to the exact values of natural frequency and modal damping.

Table 3: Numerical results for modal identification at snapshot No.1

	MODE 1		MODE 2		MODE 3	
	Freq [-]	Damp [-]	Freq [-]	Damp [-]	Freq [-]	Damp [-]
EXACT	1.00000	2.00000	1.57000	1.00000	1.96000	1.50000
CS	0.99988	1.99757	1.56996	0.99872	1.96019	1.50215
DE	0.99990	1.99984	1.57002	0.99994	1.95998	1.50290
WOA	0.99990	1.99984	1.57002	0.99994	1.95998	1.50290
GWO	0.99990	1.99984	1.57002	0.99994	1.95998	1.50290
PSO	0.99940	1.99393	0.82766	8.33283	1.96523	1.87147
FFLY	0.99979	1.99361	2.19411	7.45238	1.99272	4.83731

In order to illustrate the fitting results found, Figure 5 presents the final fitting results for the two best algorithms (DE and WOA) and two worst (PSO and FFLY).

<sup>6</sup>Larger number of significant digits were set in order to show the numerical differences.

<sup>7</sup>Modal shapes were not included because they are determined after the optimization using traditional approach, as described earlier in the present paper.

From the results it can be concluded that the algorithms that presented best performance overall during the first phase of the evaluation process were the DE and WOA. Therefore, they were selected to be used in the next evaluation phase.

Figure 6 presents the results obtained by first phase winners DE and WOA and compare them with the exact results. From the results it can be seen that there are practically no differences between the results for both natural frequencies and damping. Numerically, the highest errors found were of 0.3% in natural frequency and 1.4% for modal damping.

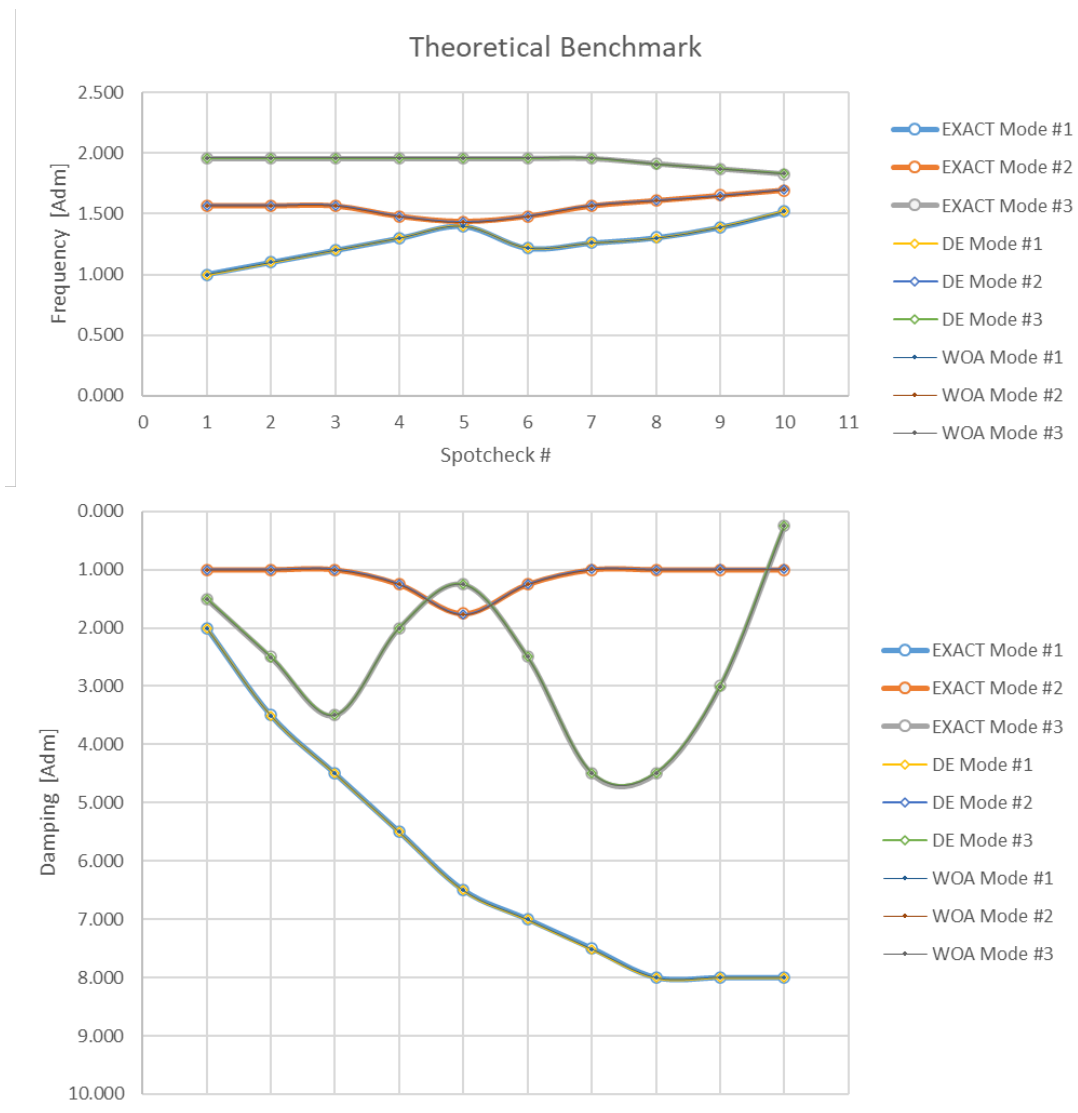


Figure 6: Theoretical benchmark, representing the aeroelastic behavior.

Both algorithms produced very similar numerical results in terms of identified natural frequency and modal damping for all three modes at the ten spotchecks. Therefore it was considered a technical draw in terms of quality of the solution produced, even for the more complicated spotchecks (#5 and #10) as illustrated by Figure 7.

On the other hand, the computational performance was quite different between DE and WOA. In average, the DE was 26% faster than WOA, being 75% faster in some spotchecks. Evidently the computational performance is affected by the initial population set-up, but the performance of DE was constantly faster than WOA.

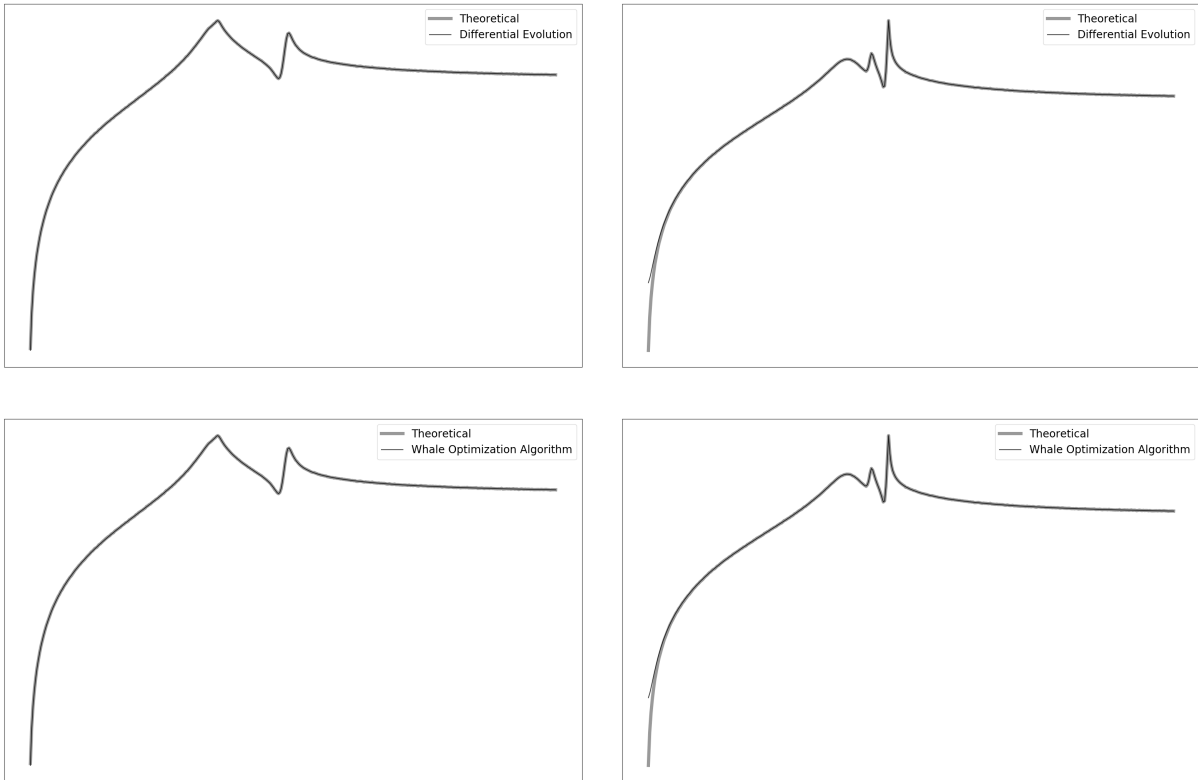


Figure 7: Final Fitting Results for the two best algorithms (DE and WOA) for most complicated spotchecks #5 and #10, respectively. FRF represented in Bode diagram, magnitude only.

From the superior performance and fine identification results produced, the DE algorithm is the best among the six evaluated algorithms for the present application.

#### 4.2 Real Flight Flutter Testing Evaluation

The experimental evaluation is based on two spotchecks from a real Flight Flutter Test campaign. These spotchecks were selected at the same flight condition and aircraft configuration, but each one focused on the identification of symmetric and antisymmetric vibration mode shapes. This strategy was employed for completeness in terms of dynamic complexity and it was considered representative and appropriate for the present application.

FRFs were estimated by using state-of-art EMBRAER process and supplied for both the Embraer Principal Frequency-Domain Algorithm (E-PFDA) and the NIA Differential Evolution (DE). Good quality FRFs were generated.

Figure 8 shows the experimental FRFs plotted with those synthesized from the identified modal parameters. It can be claimed good identification results. In particular, the DE synthesized curve is very similar to the one obtained by E-PFDA algorithm.

Table 4 presents the numerical identification results found by both, the E-PFDA and NIA DE. Eleven vibration modes were identified in the frequency range of interest. In addition, this table presents two indicators: the Natural Frequency Difference (NFD) and Natural Damping Difference (NDD), which represents the percentual differences found for the DE with respect to the E-PFDA.

NFD values lower than 1% were obtained for all identified modes, which represents a very

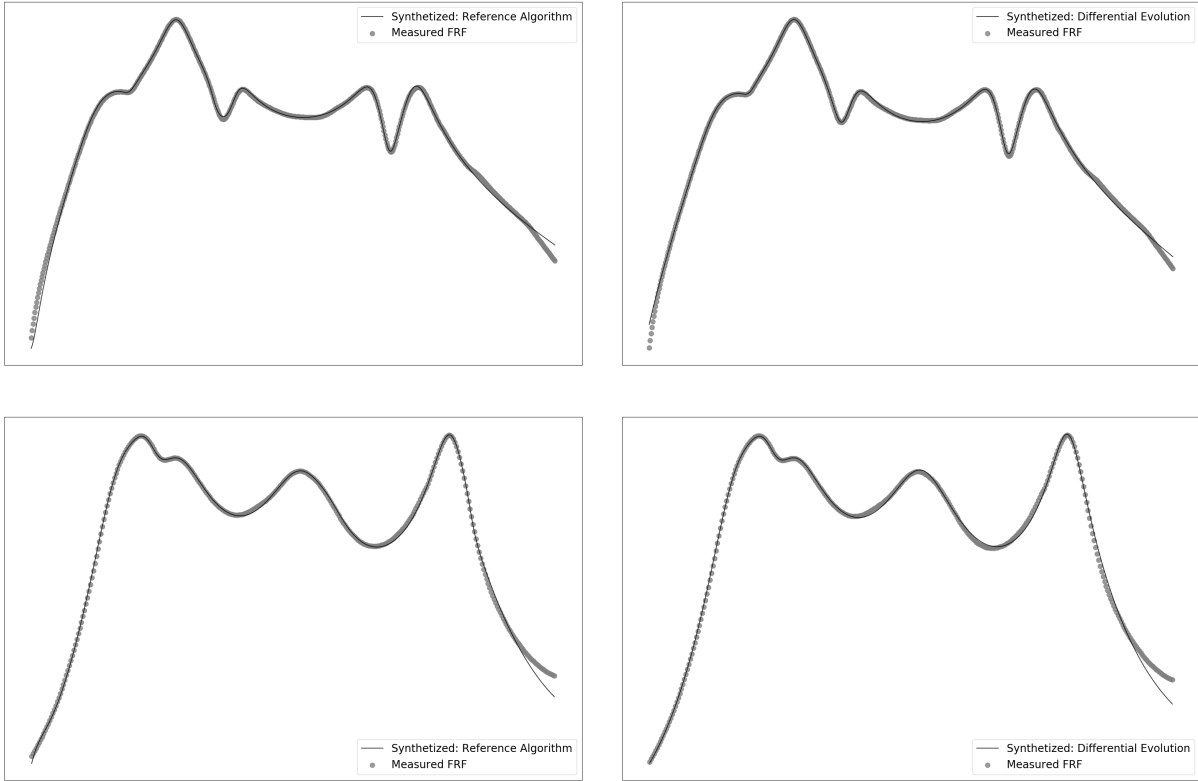


Figure 8: Experimental Fitting Results for the Embraer Principal Frequency-Domain Algorithm (E-PFDA) and the NIA Differential Evolution (DE). FRF represented in Bode diagram, magnitude only.

good performance in terms of natural frequency identification. In general, NDD lower than 8% were observed, except for mode #7 which presented 28.9 % error. The performance in terms of damping was considered good to satisfactory.

Table 4: Dimensionless numerical results for experimental modal identification by using reference algorithm E-PFDA and NIA differential evolution.

Mode #	Sym	E-PFDA		DE		INDICATORS	
		Freq [-]	Damp [-]	Freq [-]	Damp [-]	NFD[%]	NDD[%]
1	SYM	1.00	4.17	1.01	3.97	0.50	-4.76
2	ASY	1.10	2.21	1.11	2.20	0.16	-0.56
3	SYM	1.13	1.68	1.14	1.55	0.81	-7.57
4	ASY	1.19	1.44	1.19	1.47	0.05	2.22
5	ASY	1.27	2.14	1.27	2.08	-0.07	-2.61
6	SYM	1.35	1.80	1.35	1.80	-0.10	0.44
7	SYM	1.62	1.21	1.62	1.56	-0.18	28.90
8	ASY	1.60	1.95	1.61	1.89	0.53	-3.14
9	ASY	2.00	0.81	2.00	0.75	-0.04	-7.12
10	SYM	2.23	1.07	2.23	1.07	0.03	0.67
11	SYM	2.42	1.00	2.42	1.04	0.04	3.73

On the other hand, the computational performance was quite different between E-PFDA and NIA DE. In general, the E-PFDA runs in a fraction of second but the DE took minutes to solve the problem. The computational performance was considered the main drawback for industrial application of NIA DE.

## 5 CONCLUSION AND FUTURE WORK

This article proposed an innovative experimental methodology based on gradient-free optimization using Natural Inspired Algorithms. The main motivation is the use of more representative and adaptive mathematical models during the modal identification process of Flight Flutter Test campaign.

The conclusions indicate the feasibility of the NIAs to solve the nonlinear problem, with superior performance of the Differential Evolution algorithm. The results encourage a deeper research using more representative and adaptive mathematical models for the modal identification process.

Artificial Intelligence (AI) may be employed to select the most representative mathematical model structure at each test point executed, removing potential test execution restrictions (such as no test execution when turbulence level is significantly) and contributing for the increase of test campaign efficiency.

The major drawback is the computational time for finding the minimum. Better computational implementation, smarter use of initial population and parallel computation may help in this direction. To save time, a simpler analysis strategy consists in the sequential identification in narrow frequency bandwidths up to cover the target frequency range.

The possibility of using non-linear objective functions may be beneficial for optimization process. In the present paper, it was used the norm-1 instead of traditional norm-2 because it is less sensitive to noise in FRFs. Even different structures for the objective function may be explored.

Complementary, it is reasonable to assume that better NIAs may exist to solve the nonlinear problem in the modal identification process. No Physics-based methods were included herein and other Evolution-based may perform better, specially in terms of computational time. Further research is necessary and welcome in this direction.

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