

EFFICIENT AND HIGH-PRECISION NONLINEAR STATIC AEROELASTIC LOAD ANALYSIS METHOD BASED ON VORTEX LATTICE METHOD ACCELERATION

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Abstract: High-precision CFD/CSD coupling aeroelastic load analysis is an effective way to solve the nonlinear aeroelastic problem of flexible wings. In order to improve calculation efficiency while ensuring calculation accuracy, a nonlinear static aeroelastic load analysis method accelerated by vortex lattice method is proposed in this article. Vortex lattice method is adopted to achieve a faster convergence both in the iteration of static aeroelastic analysis and trimming process. An RBF-based mesh deformation tool with greedy algorithm is used to realize fast and accurate mesh deformation during the iterations. The numerical model of a commercial aircraft with very flexible wing is investigated by the traditional aeroelastic load analysis method and the accelerated method. Compared with the traditional method. The proposed accelerated method can significantly improve the calculation efficiency while ensuring the high precision.

1 INTRODUCTION

In recent years, reducing emissions and improving economic performance have become hot topics in modern commercial aircraft research. The wing aspect ratio of modern commercial transport aircraft has shown an increasing trend to enhance lift-to-drag ratios. At the same time, in order to reduce the structural weight, the proportion of composite materials in modern civil aircraft is gradually increasing. For example, the composite materials used in modern B-787 and A-350 account for 50% and 53% of the weight of the aircraft structure respectively. These designs have significantly improved the environmental economic performance of the aircrafts, but the reduced structure weight and the larger wing aspect ratio will also lead to a significant increase in the flexibility of the wing structure[1].

During flight, the wing will produce large bending and torsional deformations due to aerodynamic force, causing typical geometric nonlinear aeroelastic problems.[2] In addition, modern civil

aircraft usually fly at transonic speed, resulting in nonlinear aerodynamic forces. This makes the traditional linear analysis method unable to meet the load analysis requirements of modern civil aircraft. Meanwhile, civil aircrafts have strict safety requirements which places higher requirements on the accuracy of aeroelastic analysis. So, it is necessary to develop a new high-precision aeroelastic load analysis tool considering geometric nonlinearities effects.

Thanks to the development of computational fluid dynamics (CFD) technology and the substantial improvement in computer performance, high-precision aeroelastic analysis based on CFD/CSD coupling is becoming a research hotspot[3-5]. CFD method starts directly from the basic equations of flow, uses relatively few assumptions, simulates the essential characteristics of the flow, and can reflect the nonlinear characteristics of aerodynamic forces. However, the cost of CFD calculation is relatively high, which seriously affects the efficiency of aeroelastic calculation. It is necessary to find a way to improve calculation efficiency while ensuring the high accuracy of CFD calculations.

As a medium-precision method to calculate aerodynamic load with high efficiency, vortex lattice method (VLM) is widely used in aeroelastic analysis[6]. The VLM is derived from the potential flow equation, discretizing the wing into attached vortices distributed along the chord and span directions, and arranging free vortex lines on the trailing edge. Combing CFD with VLM, Yang et al. developed a novel way to a accelerate to convergence process of nonlinear static aeroelastic analysis[7]. CFD is used to provide high-precision aerodynamic force while VLM is used to compute the approximate incremental aerodynamic force during the Newton's iterations. While reducing the number of CFD calculations, this method can also converge to high-precision results.

Based on the work of Yang[7], this article introduces an efficient and high-precision nonlinear static aeroelastic load analysis method based on VLM acceleration. VLM is not only used in the acceleration of nonlinear static aeroelastic analysis, but also designed to achieve a faster trim convergence. A civil aircraft model with large flexible wings is analyzed by the traditional CFD/CSD coupling method and the accelerated method in this paper to verify the prove the effectiveness of the accelerated method.

2 BASIC THEORIES

2.1 Aerodynamics Analysis

Due to its high computational cost, using high precision CFD at every iterative step will lead to a sharp decrease in calculation efficiency. Meanwhile VLM is an efficient method to calculate medium precision aerodynamic force. So, the VLM is used to calculate the incremental aerodynamic force caused by deformations and CFD is used to calculate the high precision aerodynamic force.

2.1.1 CFD Theory

The high precision CFD calculation is carried out by ANSYS Fluent, which is widely used in the aerodynamic analysis in engineering calculations. In order to accurately capture the nonlinear factors such as shock wave and shock-boundary layer interference in transonic flow, the unsteady three-dimensional compressible Reynolds average N-S equation is used to calculate the aerodynamic force. The equation is

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial G}{\partial z} = J$$

where Q is the solution vector, E , F , G are flux vectors and J is the source term[8].

2.1.2 Vortex Lattice Method

In state-space form, static subsonic VLM is

$$\begin{aligned} \mathbf{A}_{\text{aero}} \mathbf{x}_{\text{aero}} + \mathbf{B}_{\text{aero}} \mathbf{w}_{\text{aero}} &= 0 \\ \mathbf{f}_A &= \mathbf{C}_{\text{aero}} \mathbf{x}_{\text{aero}} + \mathbf{D}_{\text{aero}} \mathbf{w}_{\text{aero}} \end{aligned}$$

The state variable is the strength of wake vortex rings \mathbf{x}_{aero} , with the input variables of wash \mathbf{w}_{aero} and the output variable of the aerodynamic force \mathbf{f}_A .

2.2 Structure Analysis

Traditional linear structural analysis based on the assumption of small deformation, especially the mature linear finite element method (FEM) used in traditional aeroelastic analysis, cannot accurately describe the characteristics of large deformation of large flexible aircraft. The geometrically nonlinear finite element method is basic method among the existing large deformation structure modeling theories, and has been widely used in the aeroelastic analysis of large flexible wings. This paper adopts the nonlinear FEM with updated Lagrange formulation, in which the static and kinematic variables are referenced to the configuration at the beginning of each load or time step. The equation of the updated Lagrange formulation is

$$\left({}^t\mathbf{k}_L + {}^t\mathbf{k}_{NL} \right) \mathbf{u} = {}^{t+\Delta t}\mathbf{Q} - {}^t\mathbf{F}$$

In the equation, the stiffness terms in the unit matrix are divided into linear stiffness terms ${}^t\mathbf{k}_L$ and nonlinear stiffness terms ${}^t\mathbf{k}_{NL}$. ${}^{t+\Delta t}\mathbf{Q}$ is the incremental outer force including the aerodynamic force, engine thrust, gravity at the new time step. ${}^t\mathbf{F}$ is the inner force at time step t [9]. For large deformations, aerodynamic forces are loaded as follower force to capture the changing of force directions caused by structural displacements.

2.3 Fluid-Structure Interaction

In CFD/CSD coupling aeroelastic analysis, due to the difference in the calculation methods of structural analysis and aerodynamic analysis, the structure mesh and aerodynamic mesh are usually very different. Therefore, how to transmit displacement, load and other information at the interface between the structural and aerodynamic computational domains is important in aeroelastic research.

2.3.1 Generalized Surface Spline Interpolation

To transfer loads and displacements between structural and aerodynamic models, the generalized surface spline method [10] is used in this article. The displacement interpolation relationship between structure and aerodynamic models is

$$\mathbf{U}_A = \mathbf{G}\mathbf{U}_S$$

where \mathbf{U}_A is the resulting displacements of aerodynamic mesh nodes; \mathbf{U}_S is the displacements of the structural mesh nodes and \mathbf{G} is the interpolation matrix.

Force interpolation has to satisfy the virtual work principal:

$$\delta \mathbf{U}_A^T \mathbf{F}_A = \delta \mathbf{U}_S^T \mathbf{F}_S$$

where $\delta \mathbf{U}_A^T$ and $\delta \mathbf{U}_S^T$ are arbitrary virtual displacement of aerodynamic grids and structure grids respectively. The force on structural model \mathbf{F}_S can therefore be interpolated by the aerodynamic force \mathbf{F}_A :

$$\mathbf{F}_S = \mathbf{G}^T \mathbf{F}_A$$

2.3.2 RBF-Based Mesh Deformation

During CFD/CSD coupling analysis process, the aerodynamic mesh needs to be updated after the structure deforms. Radial basis function (RBF) interpolation is widely used in mesh deformation and has been proven to be a high-precision mesh deformation method which is effective for large deformations. The basic form of radial basis function interpolation is

$$s(\mathbf{x}) = \sum_{i=1}^{N_b} \omega_i \phi(\|\mathbf{x} - \mathbf{x}_i\|)$$

In the equation, $s(\mathbf{x})$ represents the displacement value of the mesh node, N_b is the number of interpolation points, \mathbf{x}_i is the position of the interpolation node, \mathbf{x} is the position the mesh node to be calculated. ϕ is a given basis function with respect to the Euclidean distance $\|\cdot\|$. For the choice of the RBF, Wendland's C2 function is proved to have better calculation efficiency and mesh deformation quality. Its representation is

$$\phi(\xi) = \begin{cases} (1-\xi)^4 (4\xi+1) & 0 \leq \xi \leq 1 \\ 0 & \xi > 1 \end{cases}$$

where $\xi = \|\mathbf{x} - \mathbf{x}_i\| / r$ is the non-dimensional distance and r is the support radius controlling the sphere of influence of the support points[11].

For large 3-dimensional mesh, the computational cost is very high due to the large amount of nodes. Rendall et al. [12] developed an error-driven data reduction algorithm called greedy algorithm, which selects a reduced set of sample point according to the error incurred by representing the surface mesh deformation with reduced RBF interpolation. This method can significantly accelerate the process of mesh deformation calculation while ensure the quality of deformed mesh. The RBF-based mesh deformation method with greedy algorithm is used in this article for efficient and accurate mesh deformation calculation during the aeroelastic analysis.

3 LOAD ANALYSIS PROCESS

The load analysis process is based on nonlinear static aeroelastic analysis and nonlinear aeroelastic trim analysis. For the nonlinear static aeroelastic solution, the accelerated algorithm developed by Yang et al. [7] is adopted in this paper. The nonlinear iterative process of this algorithm is divided

into two processes: an inner loop based on the VLM and an outer loop based on the CFD method. In the inner loop, VLM is used to calculate the aerodynamic force to obtain an aerodynamic force closer to convergence, thereby reducing the number of CFD calculations. This method is verified by the wind tunnel test and is proved to be an effective method. The trim analysis is carried out based on the nonlinear static aeroelastic solution, and the VLM method is used to obtain the trim parameters that are close to convergence, thereby reducing the number of CFD-based trim calculations.

3.1 Accelerated Nonlinear Static Aeroelastic Analysis

The loosely coupled method is the most commonly used algorithm in CFD/CSD-based nonlinear aeroelastic analysis[13, 14]. Based on the conventional serial staggered algorithm, the nonlinear static aeroelastic analysis method accelerated by VLM is proposed. The nonlinear static aeroelastic equation is

$$\mathbf{R}_H(\mathbf{u}) = \mathbf{f}_E(\mathbf{u}) - \mathbf{f}_{AH}(\mathbf{u}) - \mathbf{f}_I = 0$$

where $\mathbf{f}_E(\mathbf{u})$ is the structure internal force, $\mathbf{f}_A(\mathbf{u})$ is the aerodynamic force and \mathbf{f}_I is the inertia force. When the system reaches the static equilibrium state, the residual satisfies $\mathbf{R}(\mathbf{u})=0$.

By introducing low-fidelity aerodynamic force calculated by VLM, we can get

$$\mathbf{R}_L(\mathbf{u}) = \mathbf{f}_E(\mathbf{u}) - \mathbf{f}_{AL}(\mathbf{u}) - \mathbf{f}_I$$

When $\mathbf{R}_L(\mathbf{u})$ converges, low-precision aerodynamic loads can be obtained and the deformed aerodynamic mesh, which is closer to the convergence state, is provided for CFD calculations. And when $\mathbf{R}_H(\mathbf{u}) = 0$, the aerodynamic loads are of high fidelity, which guarantees the accuracy of the nonlinear aeroelastic load analysis. The flowchart of the accelerated CFD/CSD coupling nonlinear aeroelastic iteration process is shown in Figure 1.

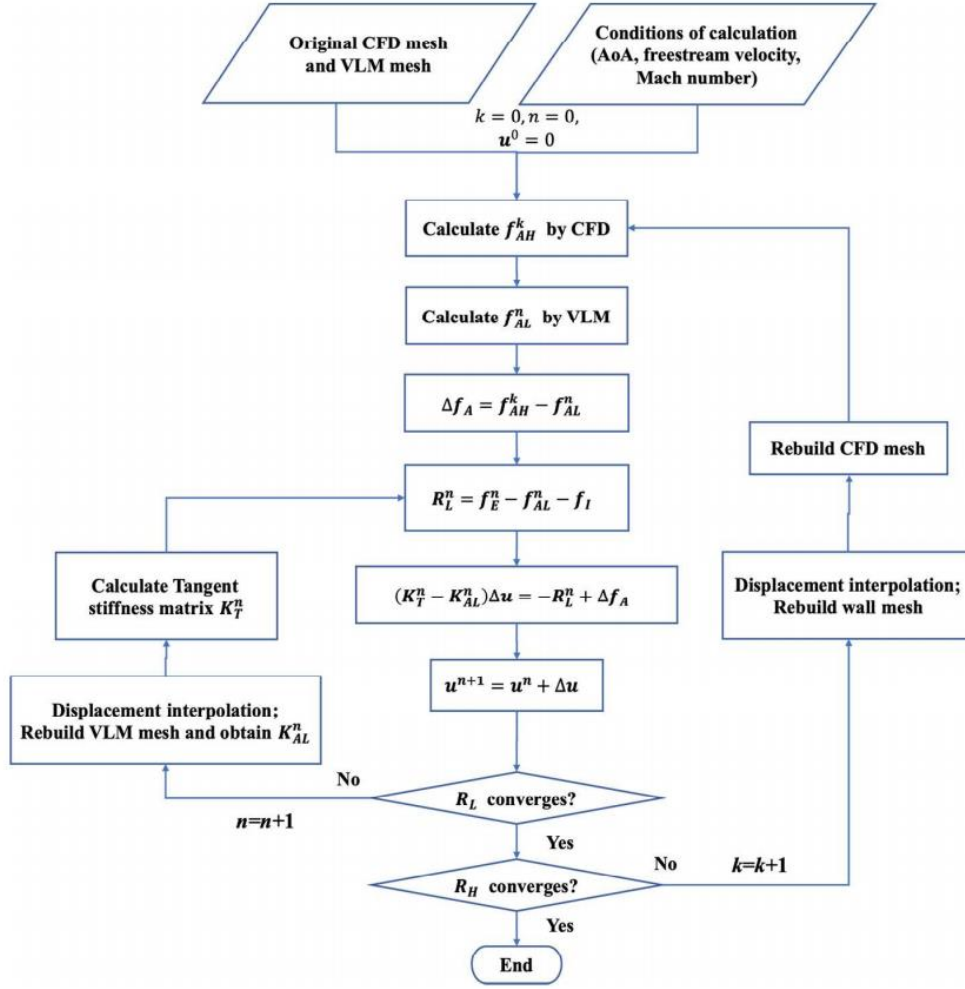


Figure 1. Accelerated CFD/CSD coupling nonlinear static aeroelastic analysis process[7]

3.2 Accelerated Nonlinear Aeroelastic Trimming Process

Trim is the prerequisite for aeroelastic load solution. For static aeroelastic trim problems, steady aerodynamic forces are used to calculate aerodynamic loads. At the same time, structural deformation is considered to be an extremely slow process, and the time derivative term of structural elastic deformation is ignored. Therefore, under the quasi-steady assumption, the aircraft nonlinear aeroelastic trimming equations can be written as

$$M\dot{V}_0 + \tilde{S}^T \dot{\omega} - M\tilde{V}_0 \omega - \tilde{\omega} \tilde{S} \omega = F_G + F_A + F_T$$

$$\tilde{S} \dot{V}_0 + J \dot{\omega} - \tilde{S} \tilde{V}_0 \omega + \tilde{\omega} J \omega = M_G + M_A + M_T$$

$$\dot{V}_0 dm + (\tilde{r}_0 + \tilde{u})^T \dot{\omega} dm + (\tilde{V}_0^T - \tilde{\omega}(\tilde{r}_0 + \tilde{u})) \omega dm + f_e(\mathbf{u}) = \mathbf{f}_G^i + \mathbf{f}_A^i + \mathbf{f}_T^i$$

Based on the equations, the VLM accelerated CFD/CSD coupling trimming algorithm is developed, as illustrated in Figure 2. The VLM based trimming process is firstly conducted to get a medium-precision trim variables close to the high precision results and deformation of the structure. Then the CFD mesh is updated and the accelerated CFD/CSD nonlinear static aeroelastic analysis is carried out. Continuously update the trim parameters in a loop until the set convergence criterion

position is met, and the geometric nonlinear trim results of the large flexible aircraft can be obtained. Based on the trim condition, the load on the aircraft can be analyzed.

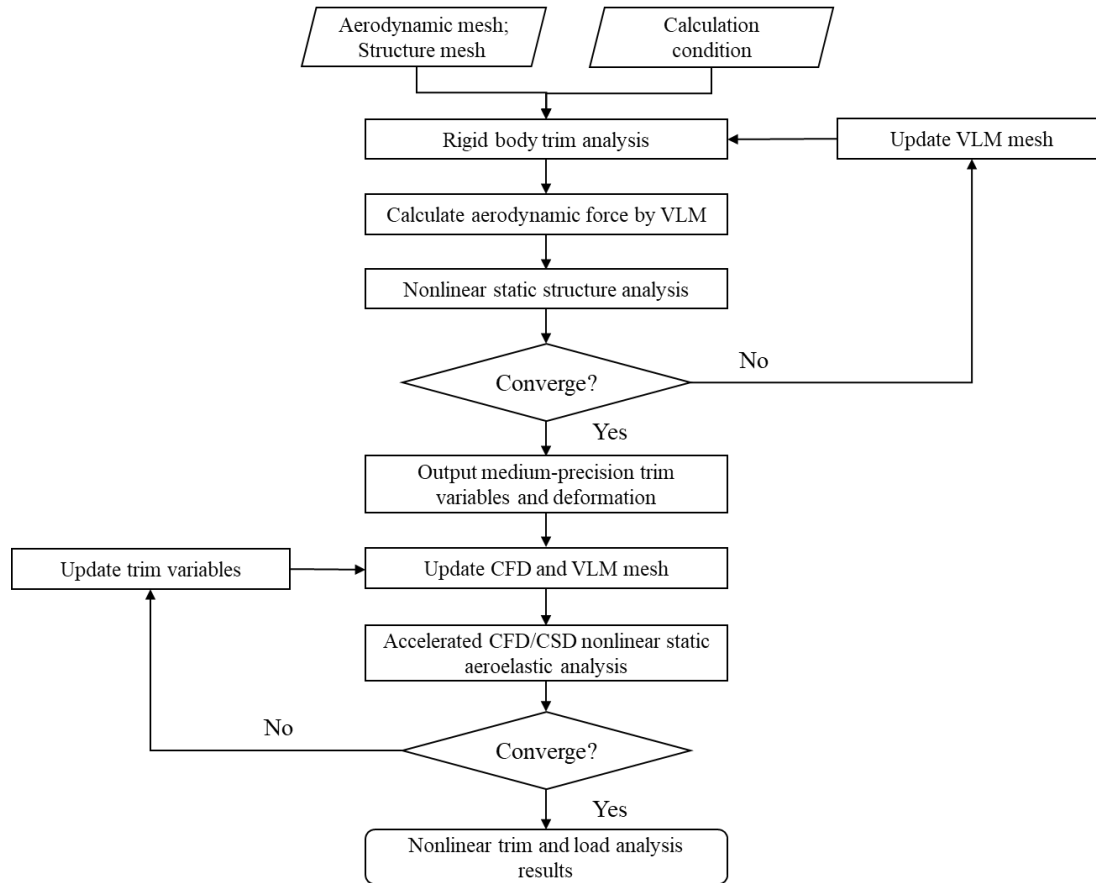


Figure 2. Accelerated CFD/CSD coupling nonlinear aeroelastic trim process

4 RESULTS

In order to verify the effectiveness of the method proposed in this article, a numerical model of civil aircraft with high aspect ratio flexible wing was analyzed using the traditional bisection trim method and the accelerated trim algorithm. The trim analysis in this article only considers angle-of-attack trim.

4.1 Numerical Model

4.1.1 Aerodynamic model

Figure 3 and 4 show the aerodynamic model of the civil aircraft with high aspect ratio flexible wing for CFD and VLM calculation. The semi wingspan of the aircraft is 2000mm. The CFD grid is a structural grid with total number of 4630268 grids. The x axis points back following the air flow. The VLM grid is built considering the camber and the preliminary twist of the wing. The coordinate system is the same with the CFD model.

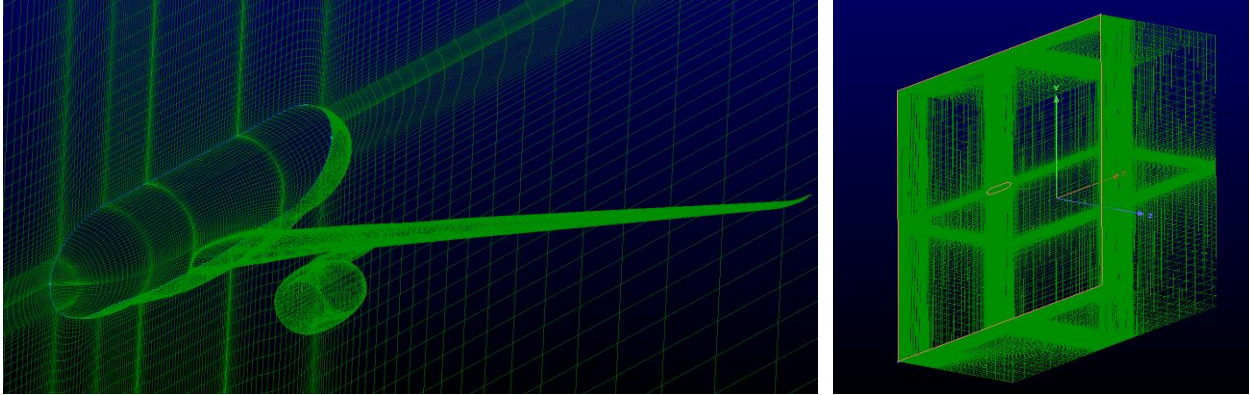


Figure 3. CFD mesh

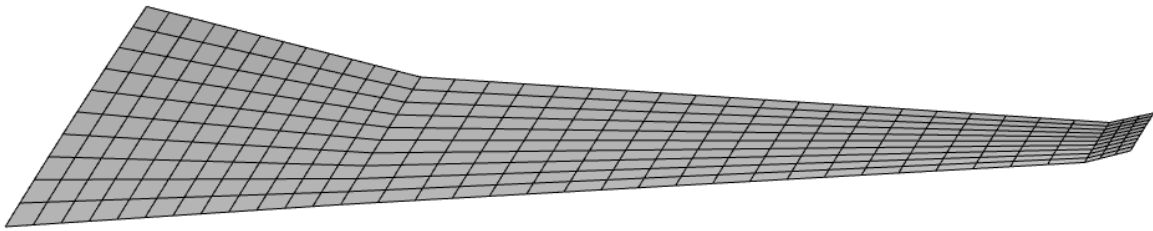


Figure 4. VLM mesh

4.1.2 Structural model

The structural model is a beam finite element model in Nastran format. The main stiffness characteristics of the wing are provided by the main beam unit, and the mass characteristics of the wing are simulated by concentrated mass points arranged at the wing ribs. The wing root is provided with fixed support constraints. The total trim structural mass of the civil aircraft model is 420kg

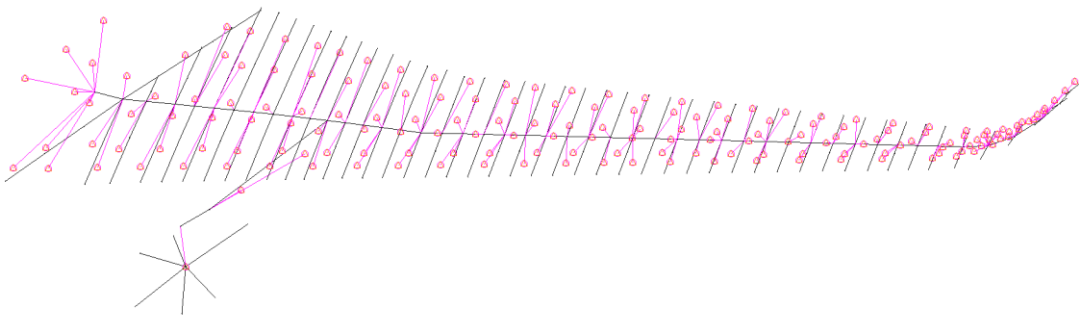
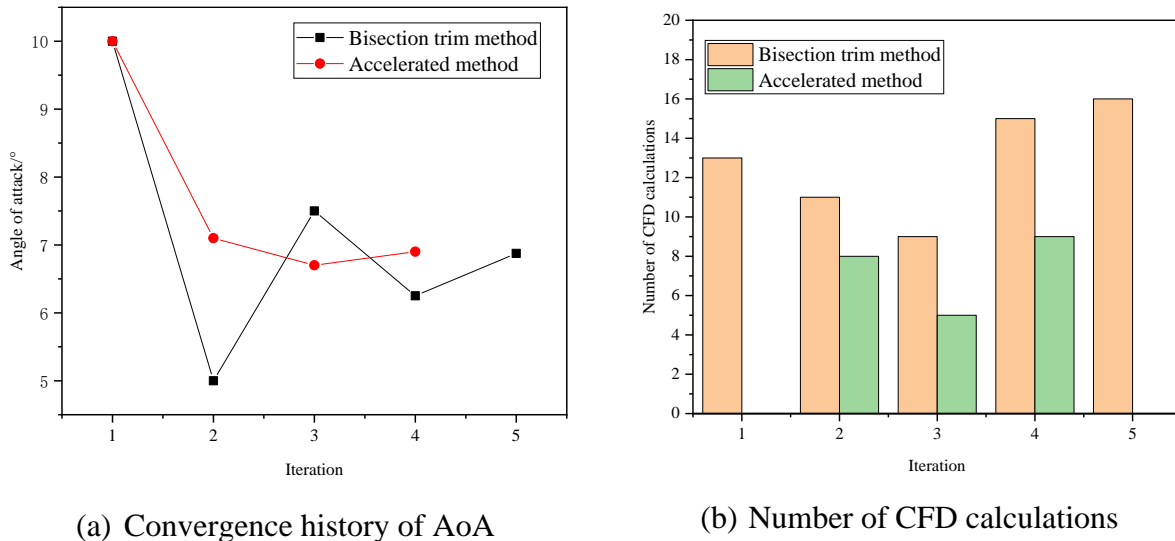


Figure 5. Structure finite element model

4.2 Load Analysis Results

Trim analysis was performed under the conditions of inflow velocity $0.6MA$ and dynamic pressure $12000PA$. The initial angle of attack for trimming is set to 10° .

Figure 6 shows the comparison of convergence history and number of CFD calculations of the traditional bisection trim method and the accelerated method. The convergence speed of the proposed accelerated method is faster than the bisection method, because the low-precision solution is provided at the beginning of the trim analysis. With the help of the nonlinear static aeroelastic analysis accelerated by VLM, the total number of CFD calculations during the trim process is reduced from 64 times to 22 times, which significantly increases the trim calculation efficiency by 65.625%. Meanwhile, the trim angle of attack is 6.9° , which is almost the same with the 6.875° calculated by the bisection method. Therefore, the proposed accelerated CFD/CSD coupling nonlinear aeroelastic trim method can improve the calculation efficiency while ensuring the same precision.

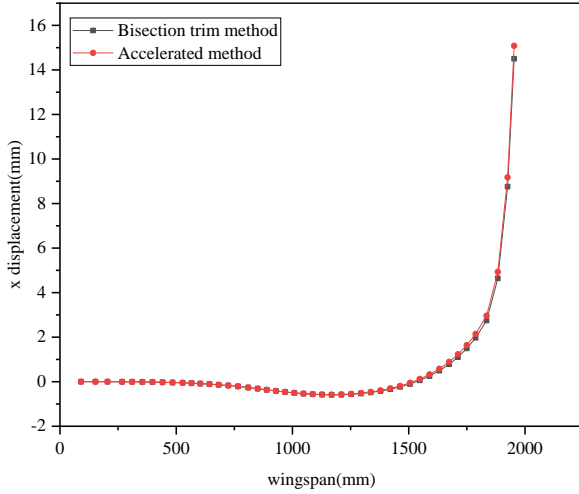


(a) Convergence history of AoA

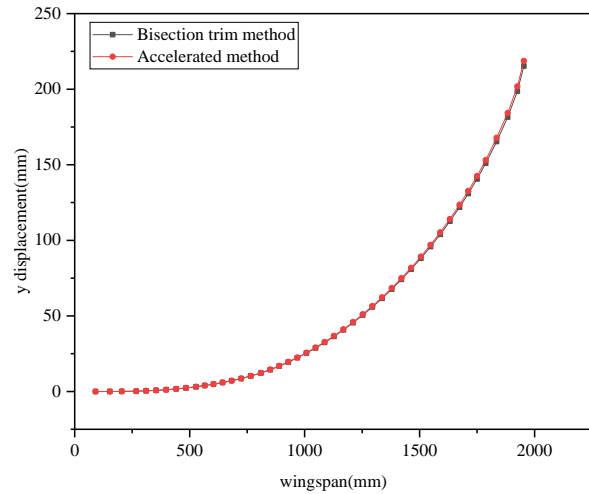
(b) Number of CFD calculations

Figure 6. Convergence history of nonlinear trim analysis

The structural deformation results of the main beam are shown in Figure 7. It can be seen that under the trimmed angle of attack, the vertical deformation of the wing tip exceeds 10% of half span. Obvious spanwise displacement is also a typical feature of geometrically nonlinear aeroelastic problems. Due to the curvature of the aerodynamic surface of the wing, the aerodynamic load has a spanwise component, which further increases the spanwise displacement of the structure under the effect of the follower load. The horizontal and vertical deformation results calculated by the two methods are almost the same, which verifies the accuracy of the load analysis.



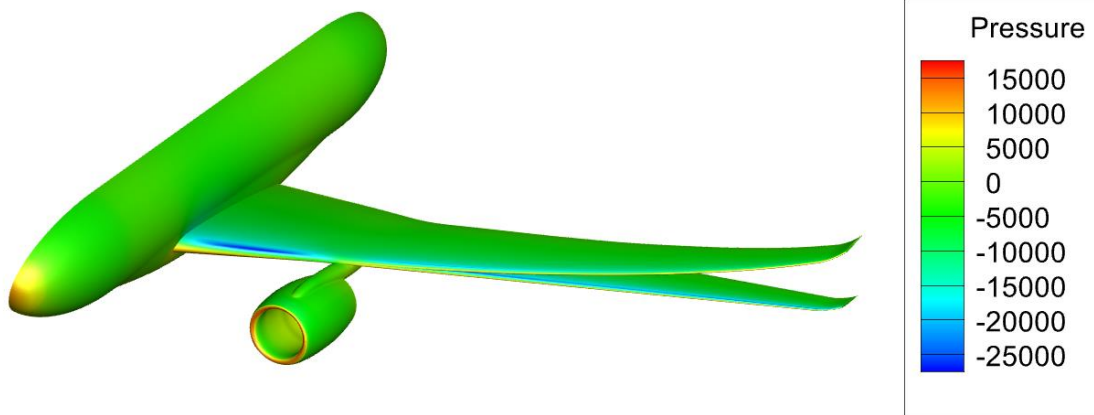
(a) Displacements in x direction



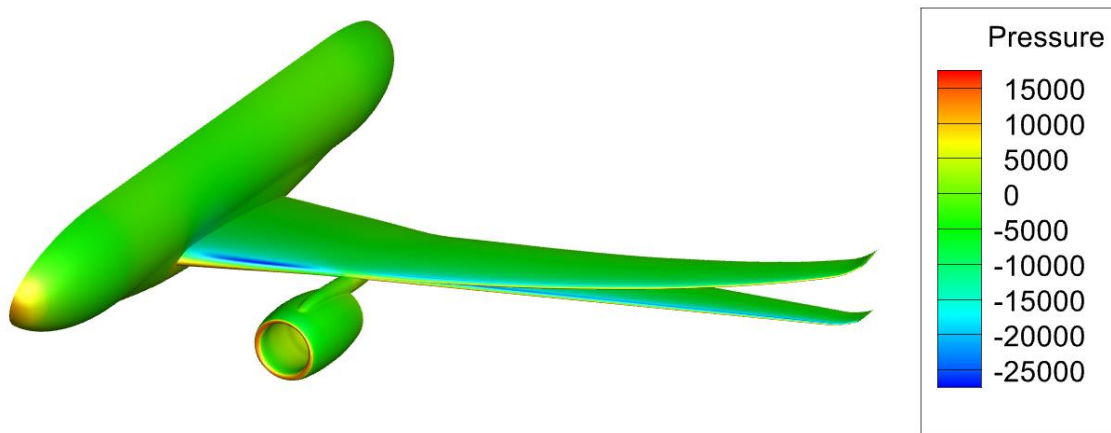
(b) Displacements in y direction

Figure 7. Comparison of structural deformation results by two methods

Figure 8 shows the trimmed distribution of static pressure on the wing. The deformed and undeformed wing are compared in the figure. The distribution of aerodynamic force changes significantly after deformation, which proves the importance of the nonlinear aeroelastic analysis. Meanwhile, the aerodynamic results of the two methods are almost the same, demonstrating the accuracy of the accelerated method.



(a) Distribution of static pressure of the bisection method



(b) Distribution of static pressure of the accelerated method

Figure 8. Comparison of distribution of static pressure results by two methods

5 CONCLUSIONS

In this article, an efficient and high-precision nonlinear static aeroelastic load analysis method based on VLM acceleration is developed to meet the aeroelastic analysis needs of modern commercial aircrafts. As a medium-precision tool with high efficiency, VLM is applied both in the static aeroelastic analysis and the trim process to get a result close to the convergence state, which can significantly reduce the consumption caused by CFD calculation. RBF-based mesh deformation tool with greedy algorithm is adopted to enable fast and accurate mesh deformation calculations. CFD is used to compute the high precision aerodynamic force and nonlinear finite element method is used for structural calculation. An airliner with a high aspect ratio flexible wing is researched to verify the accuracy and efficiency of the accelerated CFD/CSD algorithm. The results show the proposed method can converge to the same high precision results with traditional method while effectively reducing the calculation cost.

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