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ADAPTIVE EULER - EFFICIENT AND PREDICTIVE AERODYNAMICS: VALIDATION AND PROTOTYPE DEVELOPMENT TOWARD AEROELASTICITY

Johan Jansson[1], Kristoffer Wingstedt[2], Rebecca Durst[3]

[1]KTH
Stockholm, Sweden
jjan@kth.se

[2]Saab AB

[3]University of Pittsburgh

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Abstract:

We describe the Adaptive Euler methodology, and results from the High Lift Prediction Workshops with focus on the current HLPW5, showing good validation and high efficiency [74]. Adaptive Euler is first principles FEM simulation with adjoint-based adaptive error control, realized with automated discretization from mathematical notation in our FEniCS [81] framework. We describe a prototype extension of the methodology to aeroelasticity also with adjoint-based adaptive error control, such methods are highlighted as having "great potential" in the field of aeroelasticity in [80].

We show that by the Adaptive Euler by the scientific method in our reproducible Digital Math framework predicts drag, lift, pitch moment and pressure distribution in close correspondence with experiments in the 4th and 5th High Lift Prediction Workshops, with very high efficiency, estimated to 100x faster and cheaper than RANS, the industry standard for efficient aerodynamics, corresponding to appx. 100 core hours on a commodity computational resource.

Reproducibility and Open Science is described as the goal of all NASA activities in [87], we see that we realize this already today.

The guiding incentive for this work is to develop an efficient and versatile tool for aeroelasticity modeling with the Adaptive Euler methodology. Such a product is highly sought after and is motivated in part by the CFD Vision 2030 [86] set by NASA and the Certification by Analysis 2040 Vision [77] set by Boeing. The consequences of this would include–but are not limited to–the eventual development of a full fluid-structure interaction (FSI) framework that may be used for applications in aerospace engineering, including in the general Unified Continuum FSI (UC-FSI) framework we have previously developed. As such, we present numerical simulations designed to test benchmark problems in the field of aeroelasticity. These problems are chosen based on their relevance to current challenges and potential for extension, and the results are compared to experimental data when available. We view these simulations as critical building blocks towards the development of a full Adaptive Euler framework for aeroelasticity.

1 INTRODUCTION

We show that computing turbulent solutions to Euler's equations with a slip boundary condition offers a Theory of Everything ToE for slightly viscous incompressible fluid flow as a parameter-free model, covering a vast area of applications in vehicle aero/hydrodynamics including airplanes, ships and cars. This work resolves the Grand Challenges of fluid dynamics described in NASA Vision 2030.

The foundation of the methodology is an extremely efficient Direct FEM Simulation (DFS) method. We describe a breakthrough in efficiency, allowing extremely small numerical dissipation by choosing very small stabilization coefficients, while allowing very large time step size.

This work is developed as part of the Digital Math framework [1] - as the foundation of modern science based on constructive digital mathematical computation. We invite you to run and modify the simulations yourself in your web browser. The Digital Math web environment with the Open Source Adaptive Euler/FEniCS software for reproducing the results in the paper at in principle "zero" cost, together with more detailed presentation and results is available at:

http://digitalmath.tech/ifasd2024

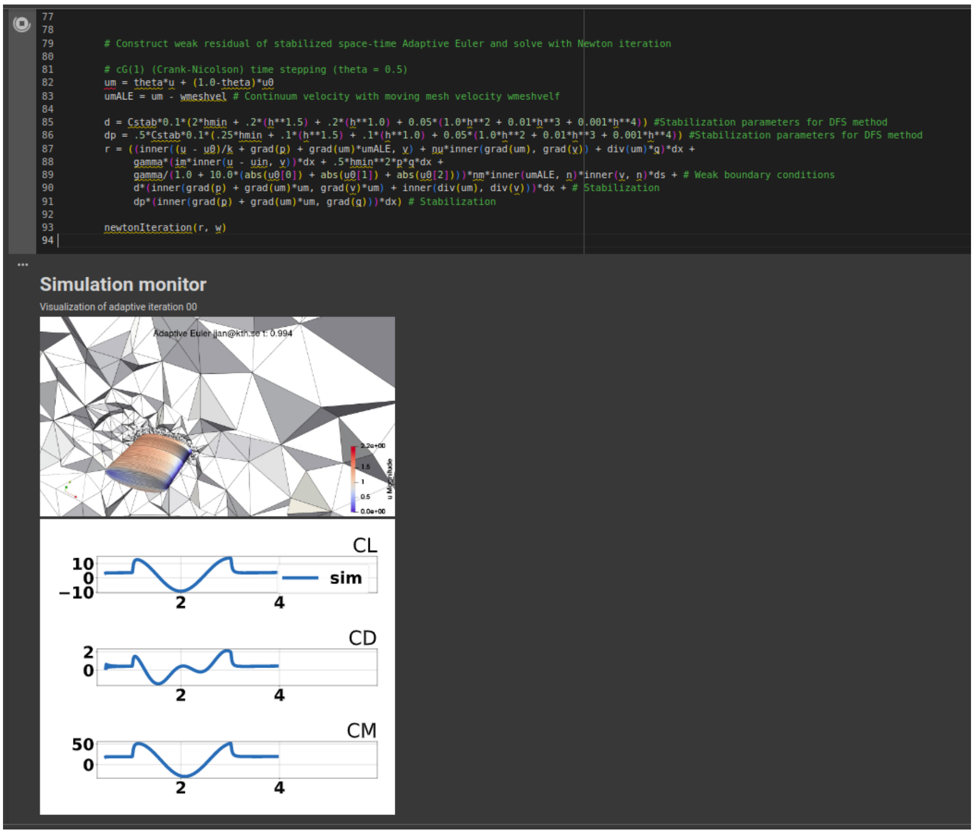
We show that Adaptive Euler by the scientific method in Digital Math predicts drag, lift and pressure distribution in close correspondence with observations for real problems with complex geometry with specific focus on the 4th High Lift Prediction Workshop and so can serve to deliver complete realistic aero/hydro-data for simulators without input from model experiments in wind tunnel and towing tank or full-scale experiments, as a new revolutionary capability.

2 ADAPTIVE EULER OVERVIEW

The methodology is a Direct FEM Simulation (DFS) [2, 4] of the first principle Euler equations with a slip boundary condition - here denoted Adaptive Euler. The methodology is realized according to the scientific method in the Digital Math framework. We call this realization Adaptive Euler.

These first principle equations are discretized by the Direct FEM approach, meaning Galerkin-Least-Squares (GLS) stabilization.

We here show a snapshot of the Digital Math environment with Adaptive Euler ALE with elastic smoothing, including the DFS formulation with GLS stabilization of the Euler equations, try yourself and test, modify, extend in your web browser on the Digital Math Adaptive Euler IFASD 2024 site http://digitalmath.tech/ifasd2024:



We see that this realizes part of Certification by Analysis 2040 [77] already today.

For clarity, the Galerkin part of the method is formulated as below in FEniCS notation:

```
F_Galerkin = inner(udot + grad(u)*u + grad(p), v)*dx
F_Galerkin += inner(div(u), q)*dx
```

and in corresponding strong form in Latex notation:

$$
\begin{aligned}
\frac{\partial u}{\partial t} + u \cdot \nabla u + \nabla p = 0 \text{ in } \Omega, \\
\nabla \cdot u = 0 \text{ in } \Omega, \\
u \cdot n = 0 \text{ on } \Gamma,
\end{aligned}
\tag{1}
$$

We have previously developed the general Unified Continuum methodology for general Fluid-Structure Interaction (UC-FSI) [82–85] , including adaptive error control. The vision is to realize Adaptive Euler in UC-FSI for general aeroelasticity.

As a first step we extend Adaptive Euler with a boundary and mesh velocity - allowing the boundary to move due to e.g. prescribed or simple rigid-body motion, this is typically called ALE. The mesh velocity simply gives rise to a convective term as compensation, denoted as "ALE" in the formulation above. Away from the boundary we are free to set an arbitrary mesh velocity. We here formulate a simple but very effective *elastic mesh smoother* - i.e. an elastic equation for the mesh velocity preserving the mesh quality for a large range of motions:

```
def epsilon(v):
  return 0.5*(grad(v) + grad(v).T)

rsmoother = inner(1.0/h*epsilon(wmeshvel), epsilon(vv))*dx + \
```

```
    gamma*nm*dot(wmeshvel - wmeshvelf, vv)*ds + gamma*im*dot(wmeshvel, vv)*dx

  newtonIteration(rsmoother, wmeshvel)
```

For completely general motions, we have also demonstrated fixed-mesh variants of UC-FSI where the marker function identifying fluid/structure is simply a sharp but continuous function, but where the efficient stabilization still appears to give what may be “good enough” efficiency with low smearing.

Another key aspect is the ability to automatically generate a “linearization” of models/equations, for automated generation of methods, eigenvalue analysis, etc.

We here demonstrate how the GLS stabilization (the key streamline diffusion term for illustrative purposes) is automatically linearized in Adaptive Euler/FEniCS, with the manual and automatic variants giving the same output to machine precision:

```
# Automated linearization using derivative
r_SD0 = derivative(d*inner(grad(u1)*u1, grad(u2)*u2)*dx, u1, u)
r_SD1 = derivative(r_SD0, u2, v)
r_SD1 = replace(r_SD1, { u1: u, u2: u })

# Manual linearization
r_SDA = r_SD1
r_SDM = d*inner(grad(u)*u, grad(v)*u)*dx
```

In [78] researchers from Dassault highlight the importance of automatic linearization and the advantages of the GLS stabilization methodology, i.e.: “The Galerkin/least-squares (GLS) formulation introduced by Hughes and Johnson, is a full space-time finite element technique ...”

An overview of the main ingredients of Adaptive Euler with the Digital Math Euler realization are given below:

Free slip boundary condition with 3D rotational slip separation

No thin boundary layers to resolve.

We show that the flow can separate with 3D rotational slip separation, at high velocity. See the 3D cylinder benchmark below for an illustration.

Our detailed validation of the reference benchmarks in the field: HiLiftPW2-4, NACA0012 wing, etc. all show that with only the pressure drag our results are within 5% of the experiment. This means skin friction drag is a small/negligible effect, which either can be omitted, or added as a minor adjustment.

In [5] we give an overview of both experimental and Euler CFD evidence, of low dependence of drag from Reynolds number in untripped configurations, consistent with free slip, from e.g. Abbott.

Automatic turbulence modeling by residual stabilisation

Through weighted least squares residual stabilisation generating *turbulent dissipation* $TD(t, u, p)$, as a solution to the open problem of *turbulence modeling* [29]. In particular, the weighted strong residual measures the *turbulent dissipation* as a mesh independent quantity meeting *Kolmogorov's K41 conjecture of finite turbulent dissipation* [73].

Adaptive adjoint-based a posteriori error control

Guaranteeing mesh-independence of drag and lift, and accuracy to a few percent in the validations.

Reproducibility

Guaranteeing the scientific method, allowing inspection, falsification, modification.

There is today a reproducibility crisis in science which we resolve with the Digital Math framework, and specifically here for Adaptive Euler.

3 DIGITAL MATH: SOLVING THE REPRODUCIBILITY CRISIS

Run and modify the simulations yourself in your web browser!

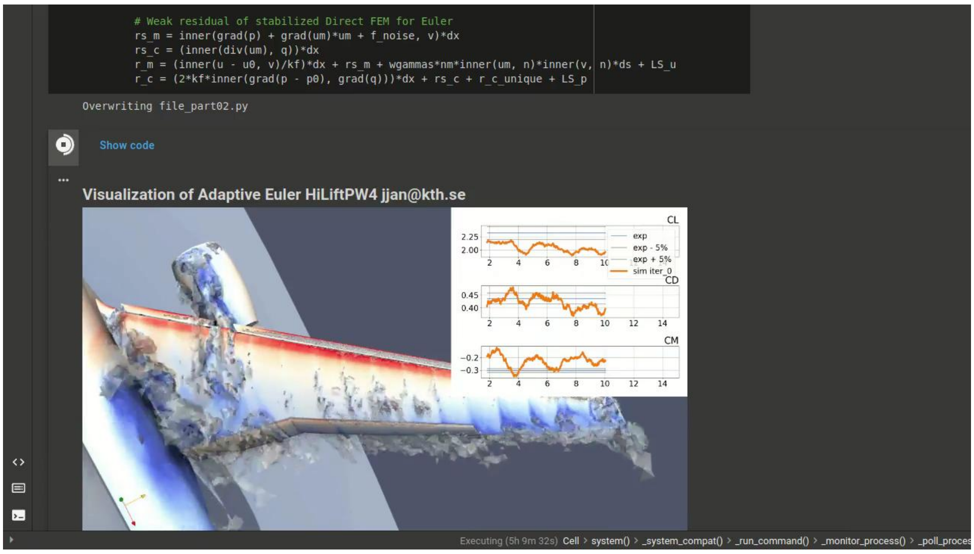
The Digital Math web environment with the Open Source Real Flight Simulator/FEniCS software for reproducing the results in the paper at in principle “zero” cost is available at:

http://digitalmath.tech/ifasd2024

together with more detailed results, articles, pedagogic material, etc.

Contact Johan Jansson (jjan@kth.se) for questions, comments and ideas.

Below is a screenshow of the Digital Math web environment with Euler Adaptive Euler interactive quasi-real-time visualization:



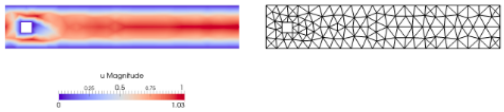
Automated Digital Math

We leverage our Open Source FEniCS framework, which automated the solution of partial differential equations by FEM, taking the mathematical notation as input, automatically generating the low-level source code without human bugs and automatically utilizing the PETSc parallel linear algebra library recognized as one of the most capable in the world. we have had long-term collaboration with the PETSc development team, giving great cross-pollination and generating what appears to be minimal abstractions. This allows an automation of Digital Math, described in a bird’s eye view below:

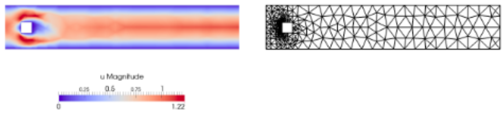
Automated discretization: (generate code for linear system from PDE/model.)

`r = (inner(grad(u), grad(v)) - inner(f, v))*dx` $\Rightarrow$ `Poisson.cpp`

Automated error control: (including parallel adaptive mesh refinement.)



$|M(e)| \leq TOL \Rightarrow$



with $M(e)$ a goal functional of the computational error $e = u - U$.

Goal: Automatically generate the **program**, **mesh** and **solution** from PDE/model (residual) and goal functional $M(U)$ (e.g. drag).

3.1 Extremely efficient stabilized Direct FEM

The predictive adaptive stabilized Direct FEM Simulation method takes the form:

```
F = F\−Galerkin + F\−Stab
```

where F_Stab is a residual-based Least Squares Galerkin stabilization of the form $(\delta R(U), R(v))$ with δ proportional to the mesh size h, controlled by the duality-based adaptive error control.

F_Stab provides numerical dissipation for the unresolved subscales in a Direct predictive and adaptive setting.

In previous work on DFS we have chosen $\delta = Ch$ or $\delta = Cmin(h)$ with $C \approx 1$. In this work we show a key breakthrough, choosing $C_U \approx 0.1$ for the velocity component U and $C_P \approx 0.01$ for the pressure component P. This corresponds to an efficiency increase of many orders of magnitude, which is key for the resolution of the Grand Challenges in fluid dynamics.

4 VALIDATION

4.1 High Lift Prediction Workshop 4

4.2 High Lift Prediction Workshop 5

5 ADAPTIVE EULER AS A SOLUTION TO NASA VISION 2030

We see that Adaptive Euler already today in 2024 satisfies the goals of the NASA Vision 2030 challenges (and partly also Certification by Analysis 2040, see relevant figures above):

1. Emphasis on physics-based, predictive modeling

Euler is predictive by being first-principles, parameter-free and mesh/discretization independent by adjoint-based adaptive error control.

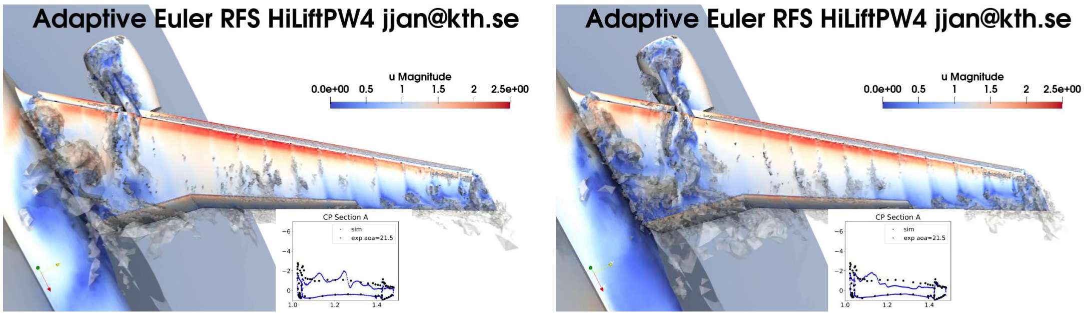


Figure 1: Digital Math Adaptive Euler simulation of the High Lift Prediction Workshop 4 benchmark, a complete aircraft with slip BC, here showing stall at aoa=21.5 . The simulation predicts the lift and drag forces of the experiment through the range of angle of attack, including stall. Adaptive error control demonstrates mesh-independence in the figures below.

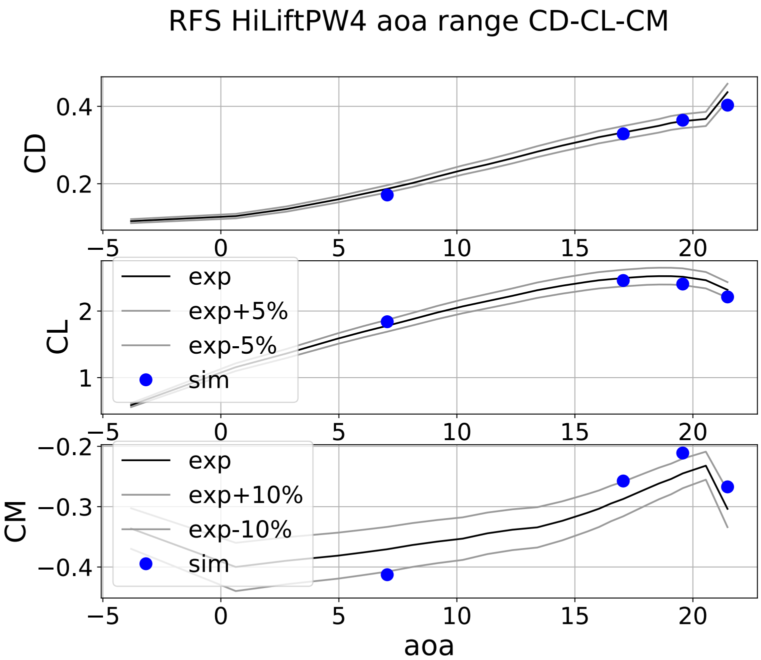


Figure 2: HLPW4 forces and pitch moment over a range of angles of attack. Prediction of CL and CD pre-stall within 5%, and CM for all angles and CL and CD at stall within 10%, specifically also predicts pitch-break.

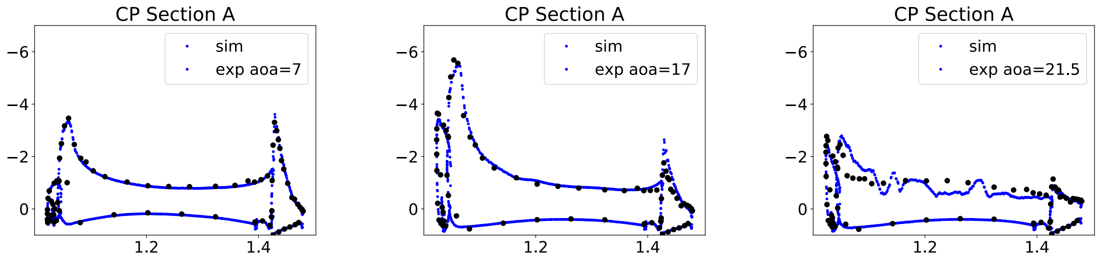


Figure 3: HLPW4 CP plots at the A stattion for aoa=7, 17 and 21.5 showing excellent match to experiment.

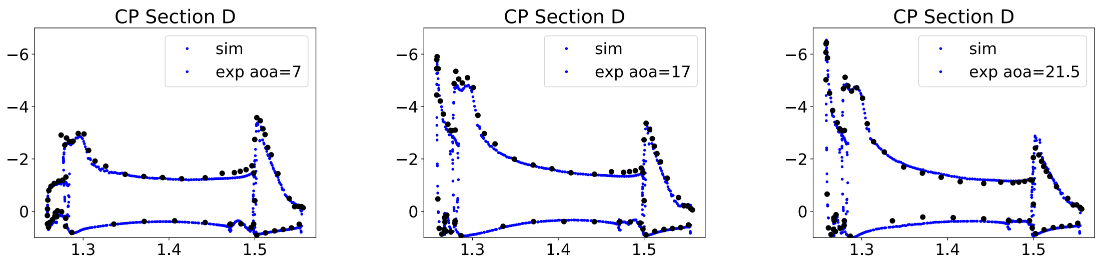


Figure 4: HLPW4 CP plots at the D stattion for aoa=7, 17 and 21.5 showing excellent match to experiment.

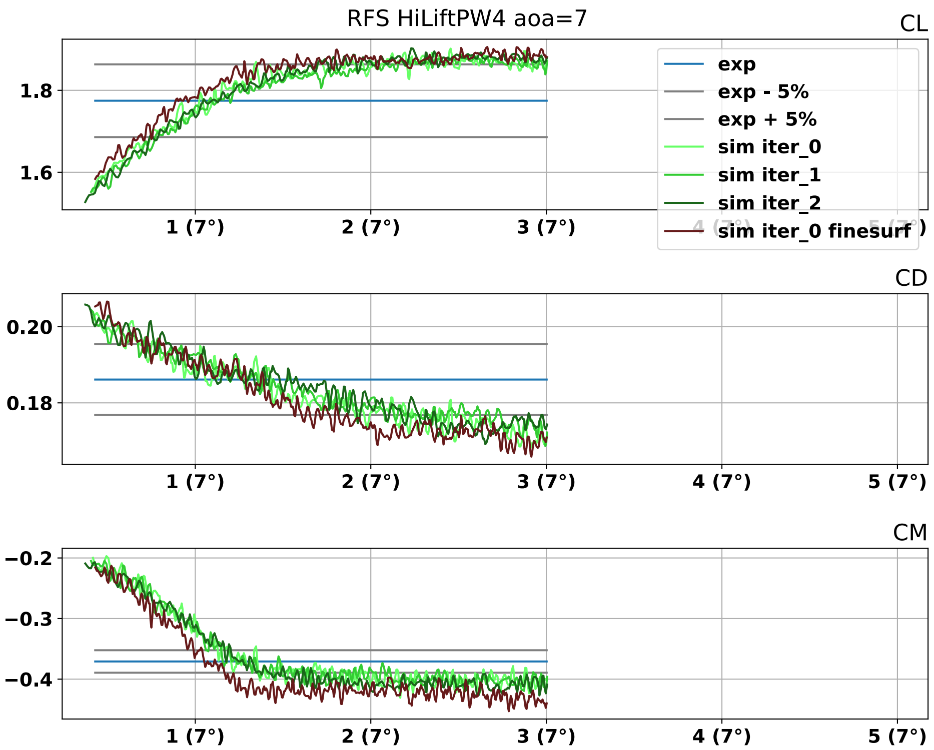


Figure 5: Digital Math simulation of the High Lift Prediction Workshop 4 benchmark, a complete aircraft with slip BC. Adaptive error control demonstrates mesh-independence, here at aoa=7. Included is also a finer surface mesh, demonstrating independence of surface mesh resolution.

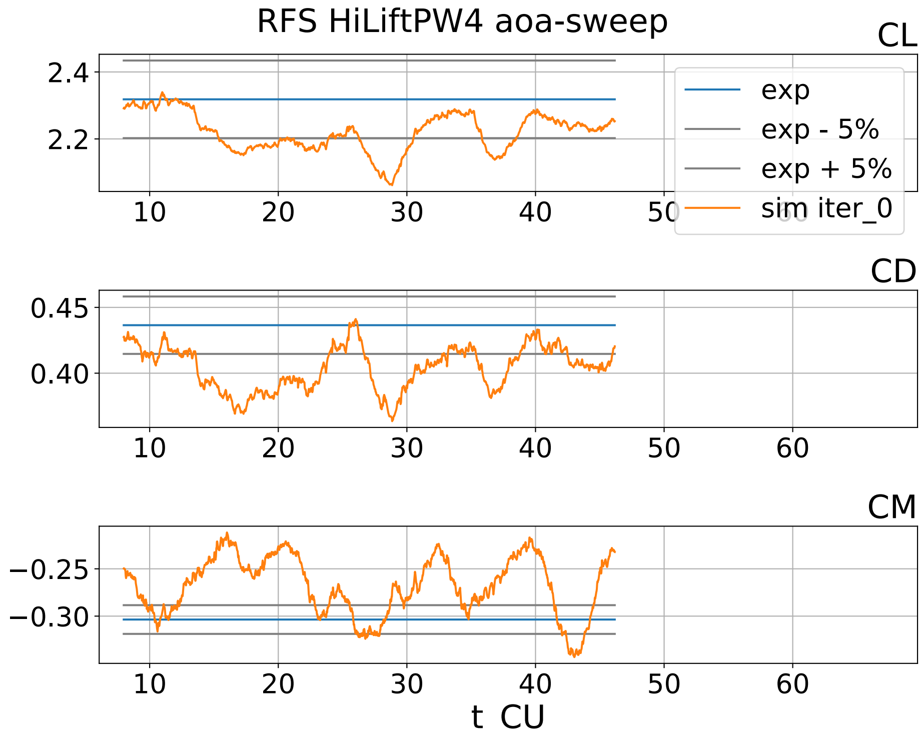


Figure 6: Digital Math simulation of the High Lift Prediction Workshop 4 benchmark, a complete aircraft with slip BC. Longer time interval at stall, t aoa=21.5 .

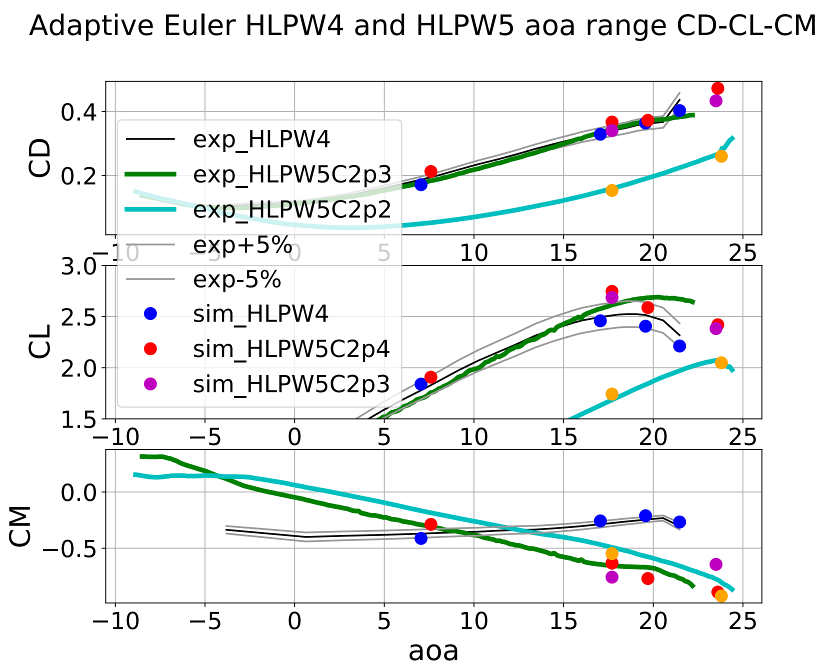


Figure 7: Preliminary HLPW5 forces and pitch moment over a range of angles of attack and three different configurations of increasing complexity. Approximately within 5% of experimental values, blind for Case 2.4. Main difference between HLPW5 an HLPW4 is the tail in HLPW5, which dominates CM which is clearly predicted by Digial Math Adaptive Euler.

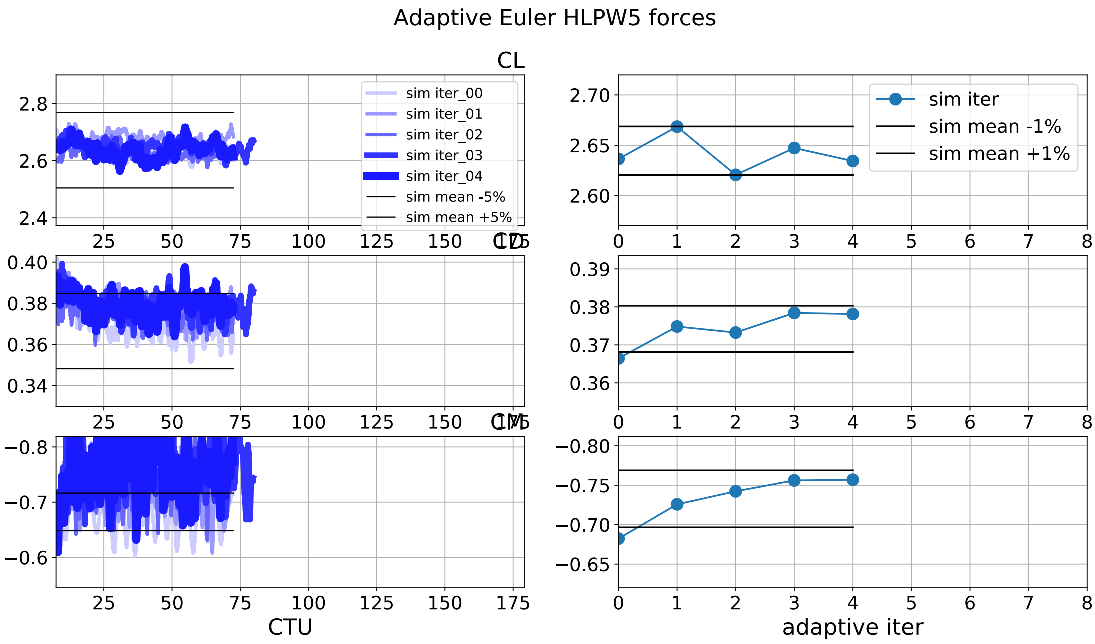


Figure 8: Preliminary HLPW5 Case 2.4 forces and pitch moment for aoa=19.7, showing mesh-independence (i.e. mesh-converged quantities) already on the extremely coarse starting mesh of 100k vertices for half-span.

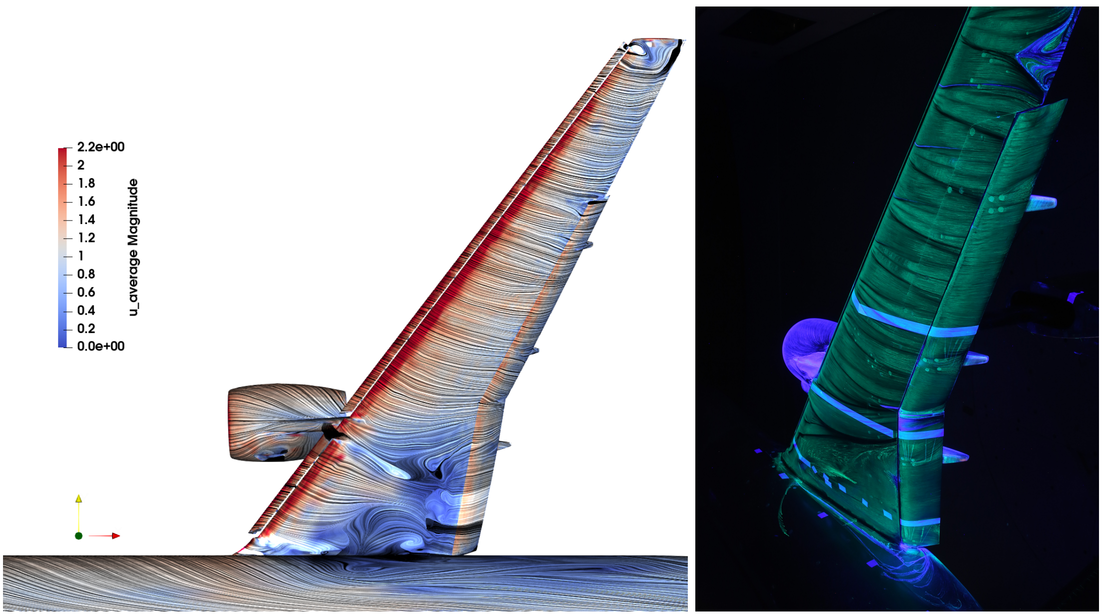


Figure 9: Preliminary HLPW5 Case 2.4 blind validation of streamlines against experimental oil flow, where Digial Math Adaptive Euler with free slip BC predicts the main separation mechanism at the wing-root juncture.

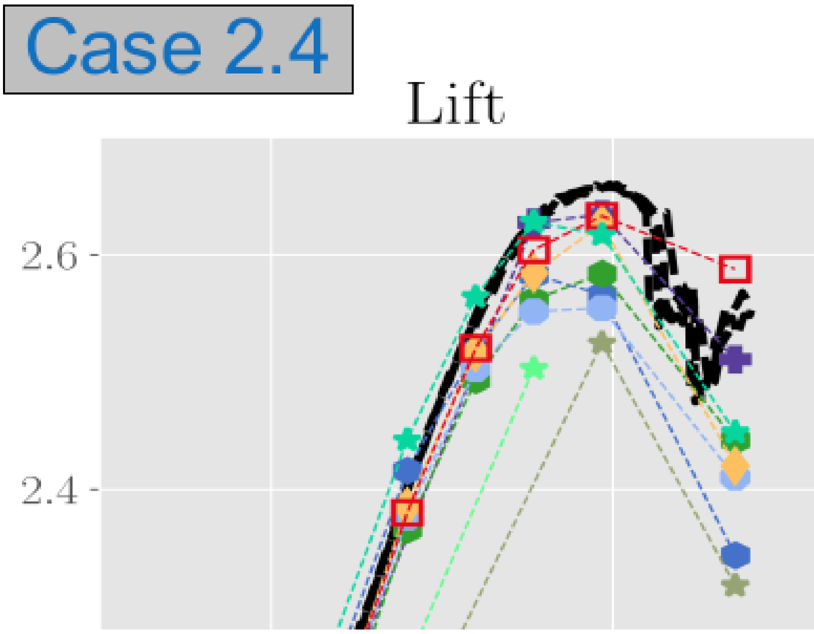


Figure 10: Preliminary HLPW5 Case 2.4 blind validation of the highest-priority CLmax quantity, where Digial Math Adaptive Euler with free slip BC predicts CLmax to within 2%, only one of three participants to do so. [88]

Computational Costs of WMLES Simulations (Case 2.4)

Method	Cells(DOF)/ Points (Millions)	Min. Spacing (mm)	Time Integ.	Time-Steps per CTU	System Type (GPU/CPU)	Number of GPUs/CPU Cores	Wall Time hours per CTU	(GPU/Core) hours per CTU	Compute Hours for 50 CTUs	Estimated Cost* ($) for 50 CTUs
WMLES (GPUs)	125	12.7	Implicit	1715	V100	120 GPUs	0.40	48	2,400	**2,520**
	325	4.0	Explicit	31,500	A100	64 GPUs	0.54	35	1,750	**2,540**
	1,260	3.2	Explicit	25,000	A10G	8 GPUs	2.25	18	900	**730**
WMLES & EULER (CPUs)	131	6.1	Implicit	2000	Milan	3840 cores	1.25	4,762	238,100	**3,850**
	131	6.1	Implicit	200	CPU	600 Cores	3.82	2,292	114,600	**1,860**
	575	1.9	Explicit	37,000	Skylake	1000 Cores	2.26	2,260	113,000	**1,830**
	0.182	25	Implicit	14	CPU	96 Cores	0.01	1	50	**0.81**
RANS** L-004	243	Grid C	Implicit	Steady State	CPU	2304 Cores	7.2 hours (2000 iterations)		16588***	**270*****

Figure 11: HLPW5 Case 2.4 simulation cost of WMLES and Euler participants, which shows that Digital Math Adaptive Euler is appx. 300x cheaper than RANS and appx. 1000x cheaper than WMLES. [88]

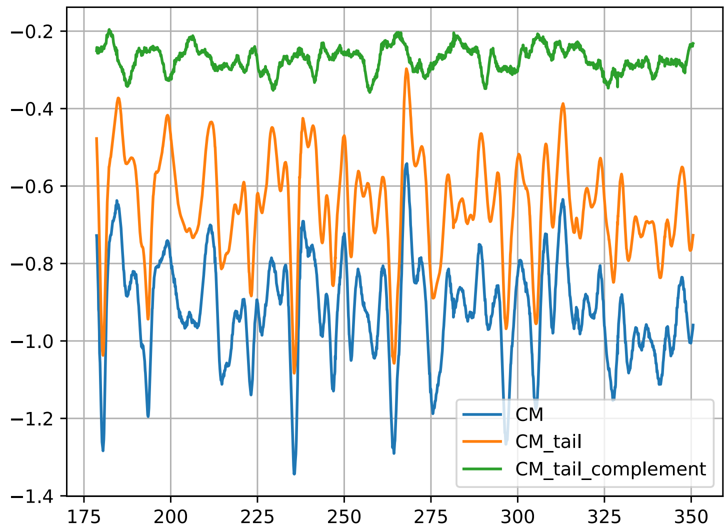


Figure 12: HLPW5 Case 2.4 for aoa=23.6 (stall angle) breakdown of contributions by component (tail and rest of aircraft) to CM, showing that the tail dominates CM and the dynamics of CM is dominated by the wake from the wing-root juncture, which we interpret as buffeting, which realizes part of Certification by Analysis 2040 already today [77]

2. Management of errors and uncertainties resulting from all possible sources

Euler is first-principles, and does not have explicit modeling parameters. Euler relies on adjoint-based adaptive error control to guarantee mesh/discretization-independence, and additionally automatically generates the low-level source code from mathematical notation (just a few lines), thus eliminating the possibility of human bugs.

3. A much higher degree of automation in all steps of the analysis process

Euler relies on automated mesh generation based on adjoint-based adaptive error control, and additionally automatically generates the low-level source code from mathematical notation (just a few lines) including the adjoint formulation and the adjoint solution.

4. Ability to effectively utilize massively parallel, heterogeneous, and fault-tolerant HPC architectures

We demontrate that Euler has extremely cheap and fast performance (200 core hours), which allows an extreme effectiveness by being able to run a large number of simulations on in principle any parallel computer (also e.g. any virtual machine in a cloud setting, even in a web browser), at a cost affordable to any engineer, researcher or even student.

5. Flexibility to tackle capability- and capacity-computing tasks in both industrial and research environments:

The same answer as above. We demontrate that Euler has extremely cheap and fast performance (200 core hours), which allows an extreme effectiveness by being able to run a large number of simulations on in principle any parallel computer (also e.g. any virtual machine in a cloud setting, even in a web browser), at a cost affordable to any engineer, researcher or even student.

6. Seamless integration with multidisciplinary analyses that will be the norm in 2030

Euler is realized in the Digital Math framework in FEniCS, taking the mathematical notation (just a few lines) as input and automatically generating the low-level source code. We have demonstrated general fluid-strucure interaction (FSI) generalizations in a very simple and automated way, and other multidisciplinary generalizations are possible or have been done in a similar way. Digital Math means an Open Source setting, where it's easy and natural to merge and integrate different formulations for e.g. different physical phenomena.

6 DIGITAL MATH: SCIENTIFIC AUTOMATED FLOW SIMULATION

The scientific process has not kept up with digital technology, and there is today a reproducibility crisis. Lorena Barba representing NASEM describes the situation as:

"The widespread use of computation and large volumes of data have transformed most disciplines of science and enabled new and important discoveries. But this revolution is not yet reflected in the ways that scientific findings are published and shared with the relevant communities. Extending the scholarly record to data, software, and computational environments and workflows is a must to ensure the robustness of science in this digital era."

We present the Digital Math framework as the foundation for modern science based on constructive digital mathematical computation, and as a solution to the reproducbility crisis. The computed result (coefficient vector, FEM function, plot, etc.) is a mathematical theorem, and the mathematical Open Source code, here in the FEniCS framework, and computation is the mathematical proof. We can also derive additional constructive proofs from the FEniCS and FEM formulation, such as stability.

Digital Math represents digitalization of science, mathematics, society and industry in the form of automated and easily understandable computation of mathematical models. It is here realized in the Open Source FEniCS framework with world-leading performance and recognized at the highest level of science and industry together with an effective pedagogical concept with combined abstract theory and mathematical interactive programming in a "one-click" cloud-HPC web-interface, accessible to anyone: In the Digital Math HiLiftPW4 site (http://digitalmath.tech/hiliftpw4-aiaa), the full adaptive Adaptive Euler methodology for several cases is available, from a cube benchmark case [3] to aircraft for you to inspect, run, modify, and reproduce, just as all examples presented. The web environment is illustrated in Figure 13.

Computational solution of turbulent solutions of Euler's equations as *Adaptive Euler* is automated using FEniCS [31] for automation of the finite element discretisation used to express the principle of best possible approximate solution. This brings a new tool of *Automated Flow Simulation* with only geometry input, which in particular allows for the first time computation of complete aero-data (forces) for any given airplane/car/ship for design, Digital Twins, interactive simulators, etc. Adaptive Euler can include (small) positive boundary friction allowing also flow before drag crisis to be computed, but it introduces friction as a coefficient to be fittted to experiments. For high Reynolds number beyond drag crisis - the regime relevant to aerodynamics - this is not needed, and Adaptive Euler is completely parameter-free.

7 PREDICTIVE ADAPTIVE EULER - RESOLUTION OF D'ALEMBERT'S PARADOX

In 1755 the German mathematician Euler formulated a mathematical model describing the flow of air (subsonic) and water with the following prophetic declaration of *Euler's Dream* [24]:

```
r_m = (inner(u - u0, v)/k)*dx + rmp_m + wgamma*nm*inner(u, n)*inner(v, n)*ds + LS_u
r_c = (2*k*inner(grad(p - p0), grad(q)))*dx + rmp_c + .5*hmin*hmin*p*q*dx + \
   (c2*dSC)*inner(grad(p), grad(q))*dx + LS_p
```

Figure 13: Digital Math Adaptive Euler simulation, visualization, editing in mathematical notation, of an electric aircraft connected to the ELISE project for electric aviation.

- *My two equations contain all of the theory of fluid mechanics. It is not the principles of mechanics we lack to pursue this analysis but only Analysis (computation), which is not sufficiently developed for this purpose.*

These are *Euler's equations* for (unit density) *slightly viscous incompressible* fluid flow formulated in terms of *fluid velocity* $u(x,t)$ and *fluid pressure* $p(x,t)$ depending on space-time coordinates (x,t) as an expression of *force balance* (Newton's 2nd Law) and *incompressibility* complemented by a *slip boundary condition*. They read [17]:

$$\begin{aligned} \frac{\partial u}{\partial t} + u\cdot\nabla u + \nabla p = 0 \text{ in } \Omega, \\ \nabla\cdot u = 0 \text{ in } \Omega, \\ u\cdot n = 0 \text{ on } \Gamma, \end{aligned} \tag{2}$$

where Ω is a spatial domain occupied by the fluid with boundary Γ with unit normal n acting like a solid wall impenetrable to the fluid as expressed by $u\cdot n = 0$. The only forces acting on the fluid (without gravitation) are the internal pressure gradient ∇p as a volume force in Ω combined with a surface pressure force pn from the wall acting in the normal direction on Γ. Formally there are no internal viscous shear forces (*zero viscosity*) and no force tangential to the boundary (*zero skin friction*).

In *bluff body flow* Ω is the domain filled by fluid flowing past a volume occupied by a solid body at rest in a coordinate system with the flow velocity being constant at large distance from the body as a far-field condition. The basic problem in bluff body flow is to determine the pressure distribution from the fluid on the body with drag and lift as net forces opposite and perpendicular to the main flow direction in normalized form appearing as coefficients of drag C_D and and lift C_L. This is the basic problem of vehicle aero/hydrodynamics including airplanes, ships and cars. We shall see that computing *turbulent solutions* of Euler's equations allows accurate prediction of drag and lift for a body of arbirtary shape, as a realisation of Euler's Dream by computing.

As is clear from (2) the Euler equations are *parameter-free* since viscosity and skin friction parameters are set to zero. This means that the Euler equations/Euler's Dream represent Einstein's ideal mathematical model as a *Theory of Everything ToE* for a certain range of physics (slightly viscous incompressible) flow, that is a mathematical theory capable of making predictions about reality (drag and lift) without any input of parameters such as viscosity and skin friction. We give below massive evidence that computation of turbulent solutions to Euler's equations is a ToE for fluid mechanics, and as such very remarkable and useful. But it took 250 years to make computing powerful enough to make Euler's Dream come true, and the start for Euler in 1755 was rocky.

Eulers French adversary mathematician d'Alembert namely quickly crushed Euler's grand plan by showing that Euler's equations admitted certain solutions (potential solutions) showing zero drag and lift of a body moving through air or water, in direct contradiction to observation [23, 30, 33, 34]. This was coined *d'Alembert's Paradox* (in fact realised by Euler before 1755 [34]), which from start as expressed by Chemistry Nobel Laureate Hinshelwood, *separated practical fluid mechanics (hydraulics) describing phenomena (drag, lift), which cannot be explained, from theoretical fluid mechanics explaining phenomena (zero drag, lift), which cannot be observed.*

We show that predictive Adaptive Euler resolves the paradox. The potential solution with zero drag is unstable - shown by stability analysis and computational evidence with adaptive error control. We illustrate the resolution by the basic cylinder model problem, and also by the most advanced benchmark in the world representing vehicles and aerodynamic devices - the High Lift Prediction Workshop, where we show that Euler CFD predicts the experiment to 5% with mesh independence, and predicts they key stall mechanism.

In Figure 14 we show our resolution of the paradox with Digital Math Adaptive Euler: the potential solution is unstable and develops streamwise vortices on the downstream side of the cylinder, generating "3D slip separation" - separation at high flow velocity.

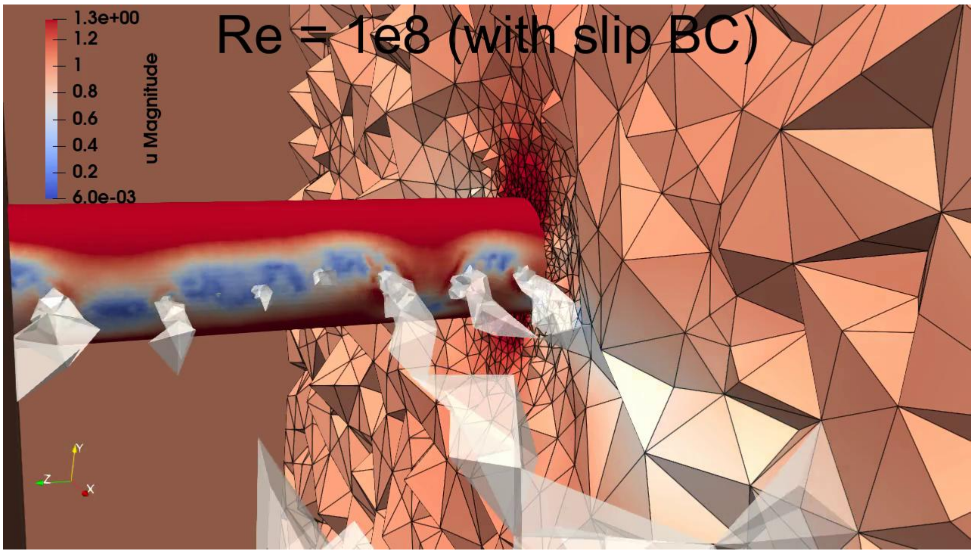


Figure 14: Digital Math simulation of a 3D cylinder with slip BC, and a sweep over the Reynolds number/viscosity. Starting at Re=1e9 (low viscosity), we observe the "3D slip separation" with streamwise vortices generated on the downstream side of the cylinder. However, by Re=1e3 and Re=1e4 this mechanism is damped out.

In the HiLiftPW4 results throughout this article, we show prediction of stall, both CD and CL within 5% of the experiment and mesh-independent. This represents the resolution of the NASA Vision 2030 grand challenge, and with Digital Math guaranteeing the scientific method.

8 CONCLUSIONS

We have showed that computing turbulent solutions to Euler's equations with a slip boundary condition offers a Theory of Everything ToE for slightly viscous incompressible fluid flow as a parameter-free model, we are now able to predict a vast area of applications in vehicle aero/hydrodynamics including airplanes, ships and cars. This work resolves the Grand Challenges of fluid dynamics described in NASA Vision 2030.

Key specific results are breakthrough results validating Euler for the High Lift Prediction Workshops and rigid-body aeroelastic wing cases.

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